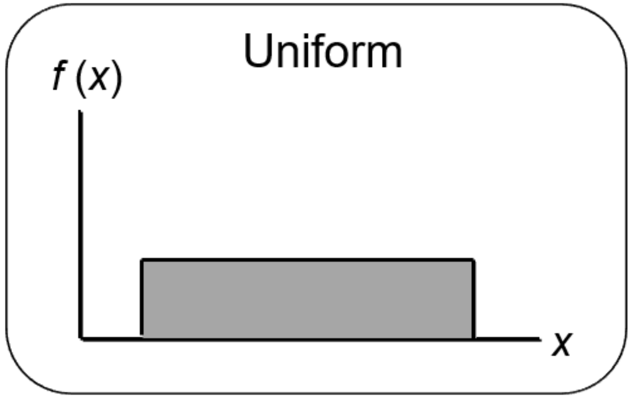
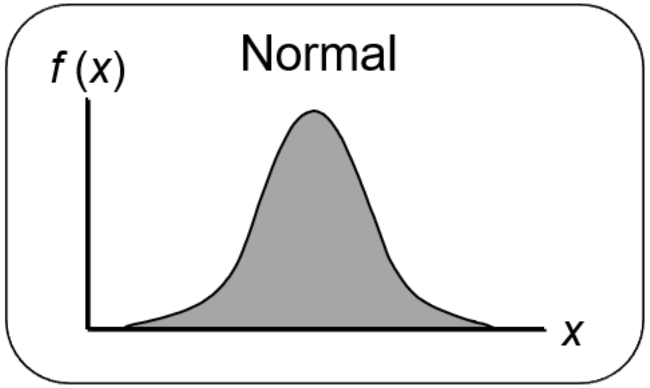
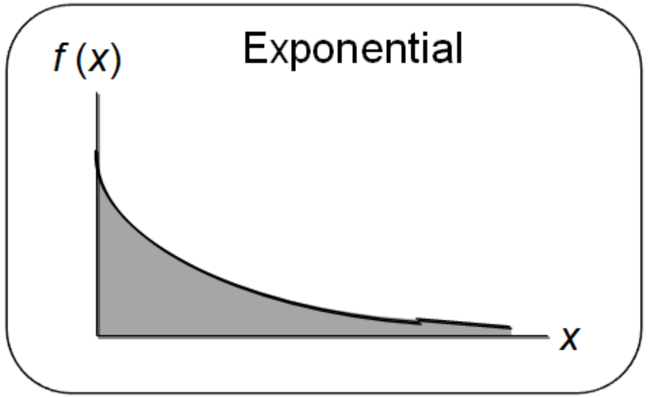
Chapter 6
Continuous Probability Distributions

- Uniform Probability Distribution
- Normal Probability Distribution
- Exponential Probability Distribution





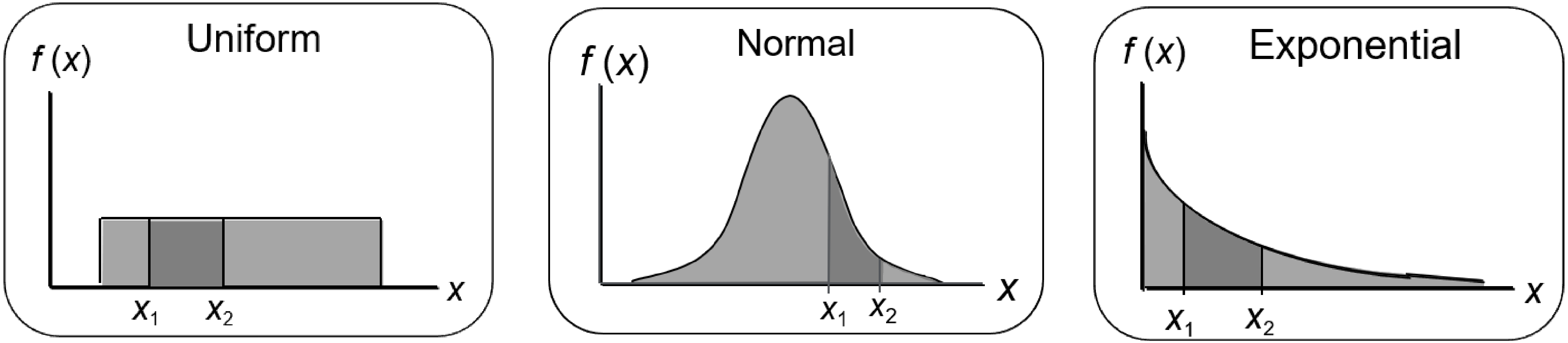


Continuous Probability Distributions (1 of 2)

- A continuous random variable can assume any value in an interval on the real line or in a collection of intervals.
- It is not possible to talk about the probability of the random variable assuming a particular value.
- Instead, we talk about the probability of the random variable assuming a value within a given interval.

Continuous Probability Distributions (2 of 2)

- The probability of the random variable assuming a value within some given interval from x_1 to x_2 is defined to be the area under the graph of the probability density function between x_1 and x_2.



Uniform Probability Distribution (1 of 7)

- A random variable is uniformly distributed whenever the probability is proportional to the interval's length.
- The uniform probability density function is:

$$f(x) = 1/(b - a) \text{ for } a \leq x \leq b$$
$$= 0 \text{ elsewhere}$$

where: *a* = smallest value the variable can assume

b = largest value the variable can assume

Uniform Probability Distribution (2 of 7)

- Expected Value of *x*

$$E(x) = (a + b)/2$$

- Variance of *x*

$$\text{Var}(x) = (b - a)^2/12$$

Uniform Probability Distribution (3 of 7)

- **Example:** Flight time of an airplane traveling from Chicago to New York
  - Suppose the flight time can be any value in the interval from 120 minutes to 140 minutes.

Uniform Probability Distribution (4 of 7)

- Uniform Probability Density Function

$$f(x) = 1/20 \text{ for } 120 \le x \le 140$$
$$= 0 \text{ elsewhere}$$

where:

x = Flight time of an airplane traveling from Chicago to New York

Uniform Probability Distribution (5 of 7)

- Expected Value of x

$$\begin{aligned} E(x) &= (a+b)/2 \\ &= (120+140)/2 \\ &= 130 \end{aligned}$$

- Variance of x

$$\begin{aligned} Var(x) &= (b-a)^2/12 \\ &= (140-120)^2/12 \\ &= 33.33 \end{aligned}$$

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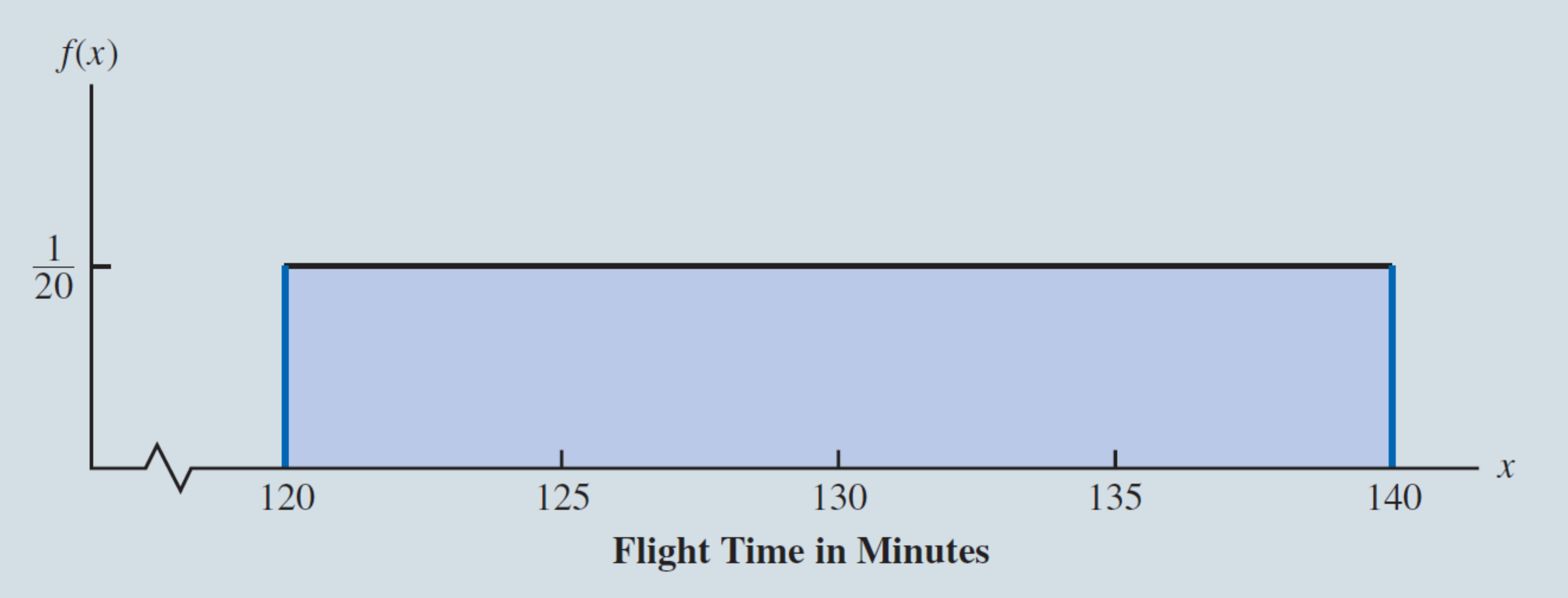
Uniform Probability Distribution (6 of 7)

- **Example:** Flight time of an airplane traveling from Chicago to New York

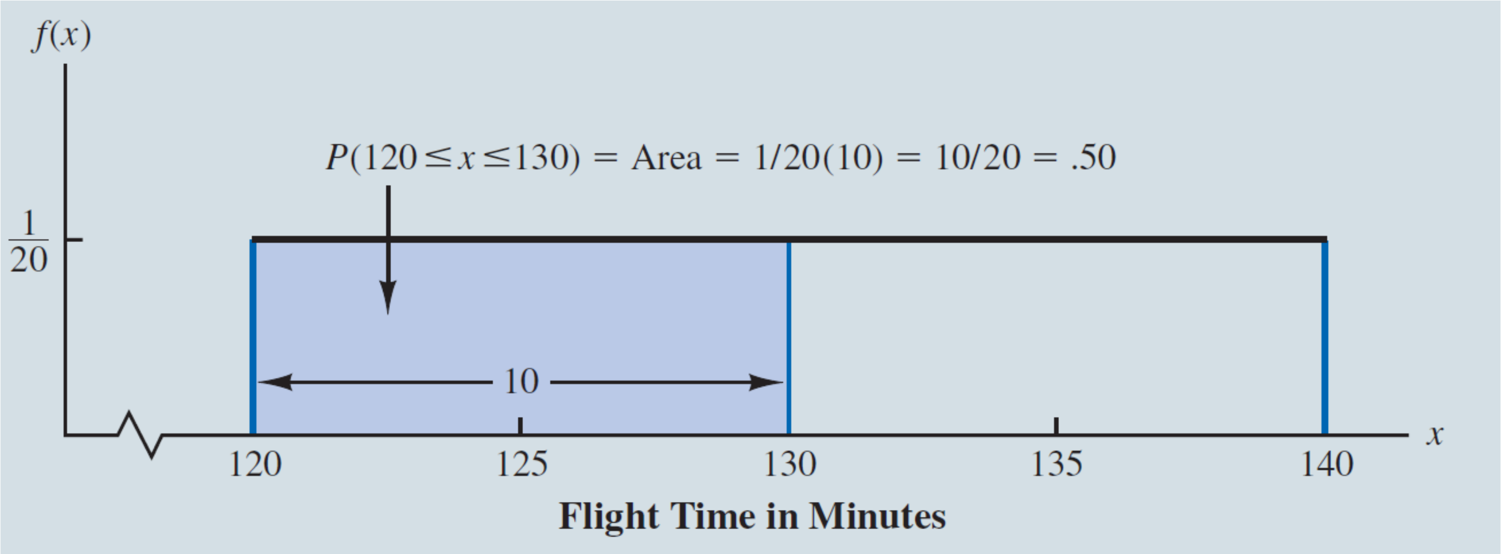


Uniform Probability Distribution (7 of 7)

- **Example:** Flight time of an airplane traveling from Chicago to New York

  Probability of a flight time between 120 and 130 minutes

$$P(120 \le x \le 130) = 1/20(10) = .5$$



Area as a Measure of Probability

- The area under the graph of *f(x)* and probability are identical.
- This is valid for all continuous random variables.
- The probability that x takes on a value between some lower value x_1 and some higher value x_2 can be found by computing the area under the graph of *f(x)* over the interval from x_1 to x_2.

Normal Probability Distribution (1 of 10)

- The normal probability distribution is the most common distribution for describing a continuous random variable.
- It is widely used in statistical inference.
- It has been used in a wide variety of applications including:
  - Heights of people
  - Test scores
  - Rainfall amounts
  - Scientific measurements
- Abraham de Moivre, a French mathematician, published *The Doctrine of Chances in 1733*. He derived the normal distribution.

Normal Probability Distribution (2 of 10)

- Normal Probability Density Function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Where μ = mean

σ = standard deviation

π = 3.14159

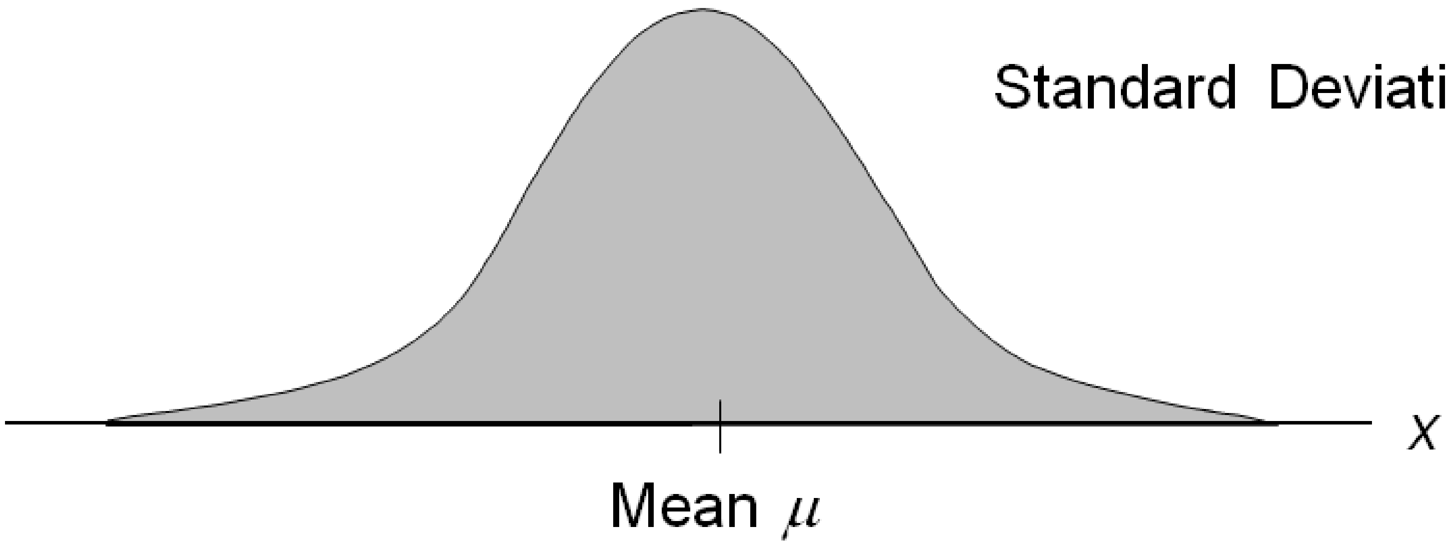
e = 2.71828

Normal Probability Distribution (3 of 10)

- Characteristics
  - The distribution is symmetric; its skewness measure is zero.

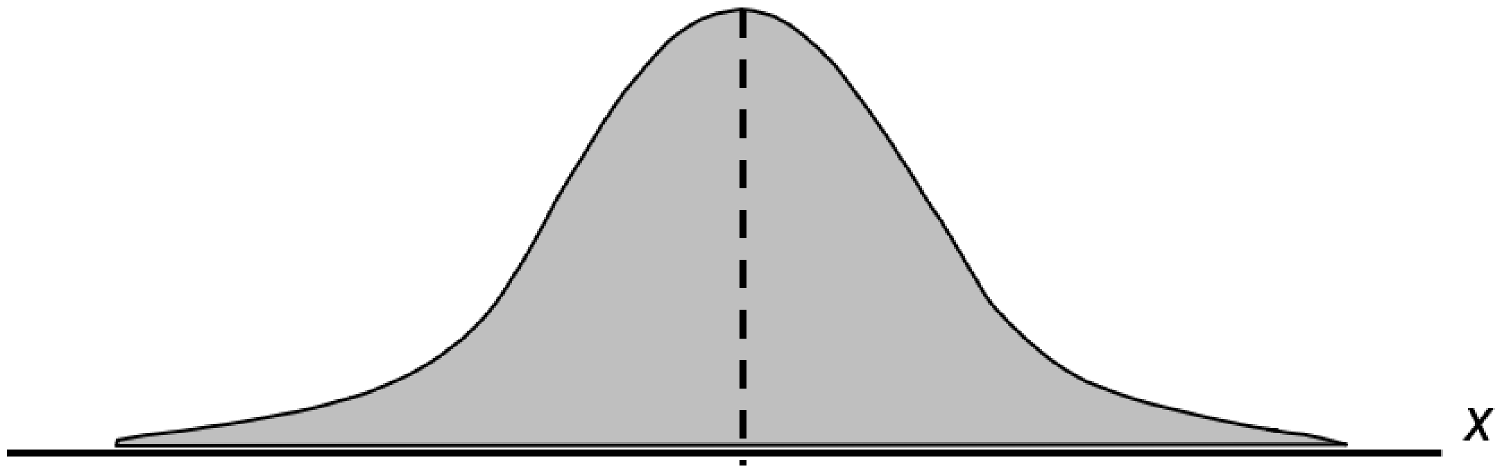
Normal Probability Distribution (4 of 10)

- Characteristics
  - The entire family of normal probability distributions is defined by its mean μ and its standard deviation σ.



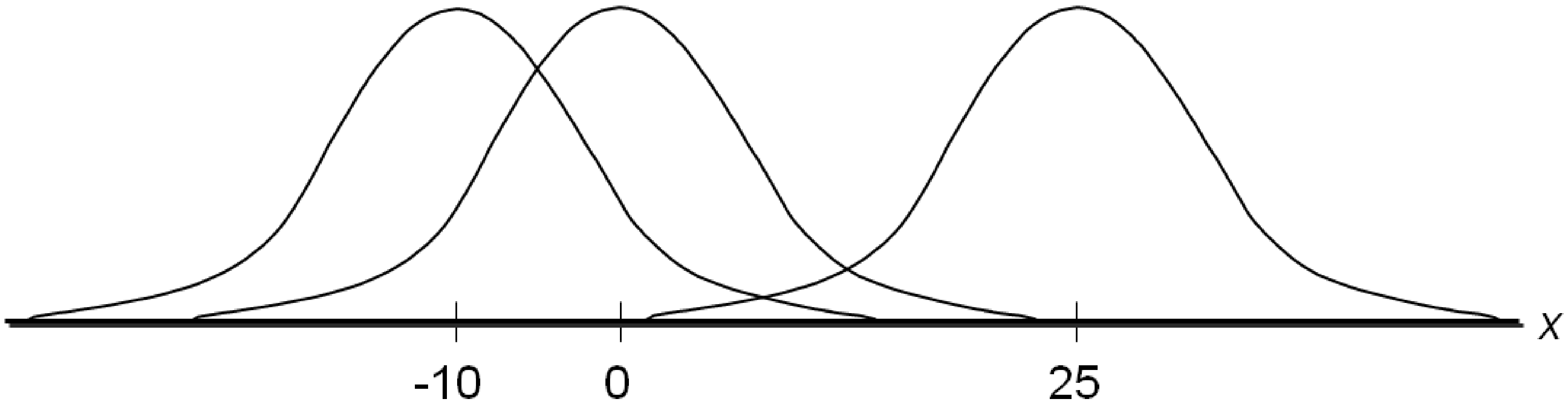
Normal Probability Distribution (5 of 10)

- Characteristics
  - The highest point on the normal curve is at the mean, which is also the median and mode.



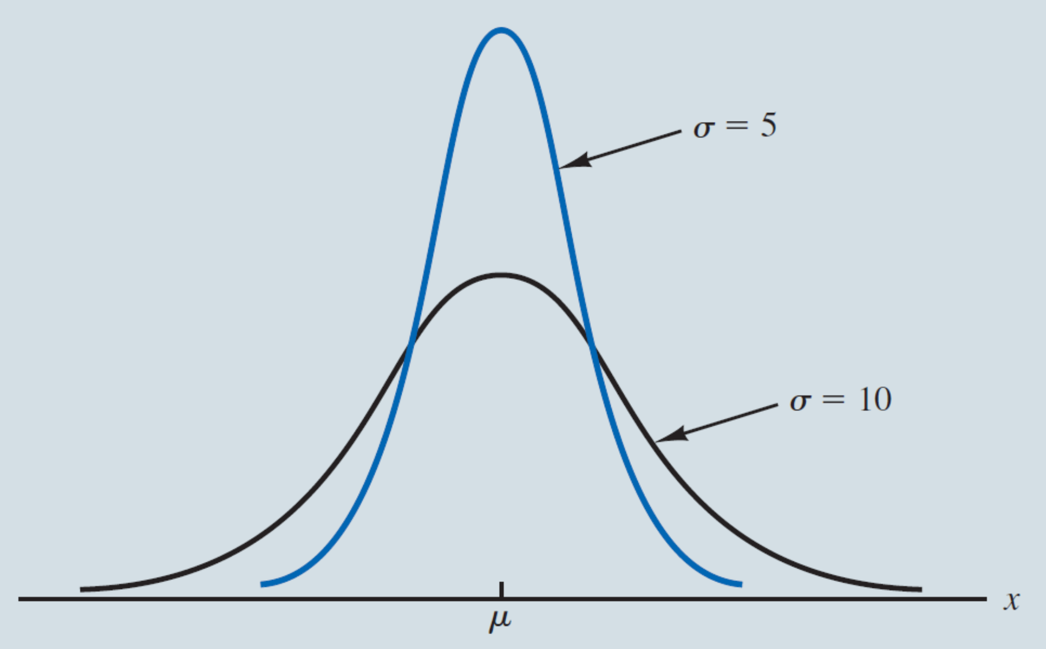
Normal Probability Distribution (6 of 10)

- Characteristics
  - The mean can be any numerical value: negative, zero, or positive.



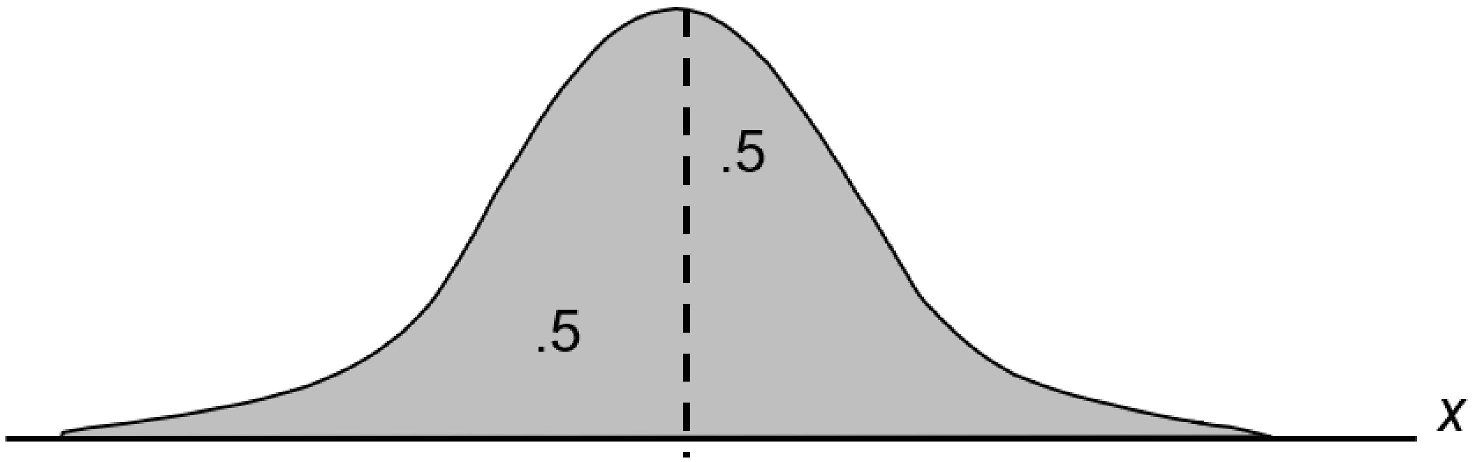
Normal Probability Distribution (7 of 10)

- Characteristics
  - The standard deviation determines the width of the curve: larger values result in wider, flatter curves.



Normal Probability Distribution (8 of 10)

- Characteristics
  - Probabilities for the normal random variable are given by areas under the curve. The total area under the curve is 1 (.5 to the left of the mean and .5 to the right).

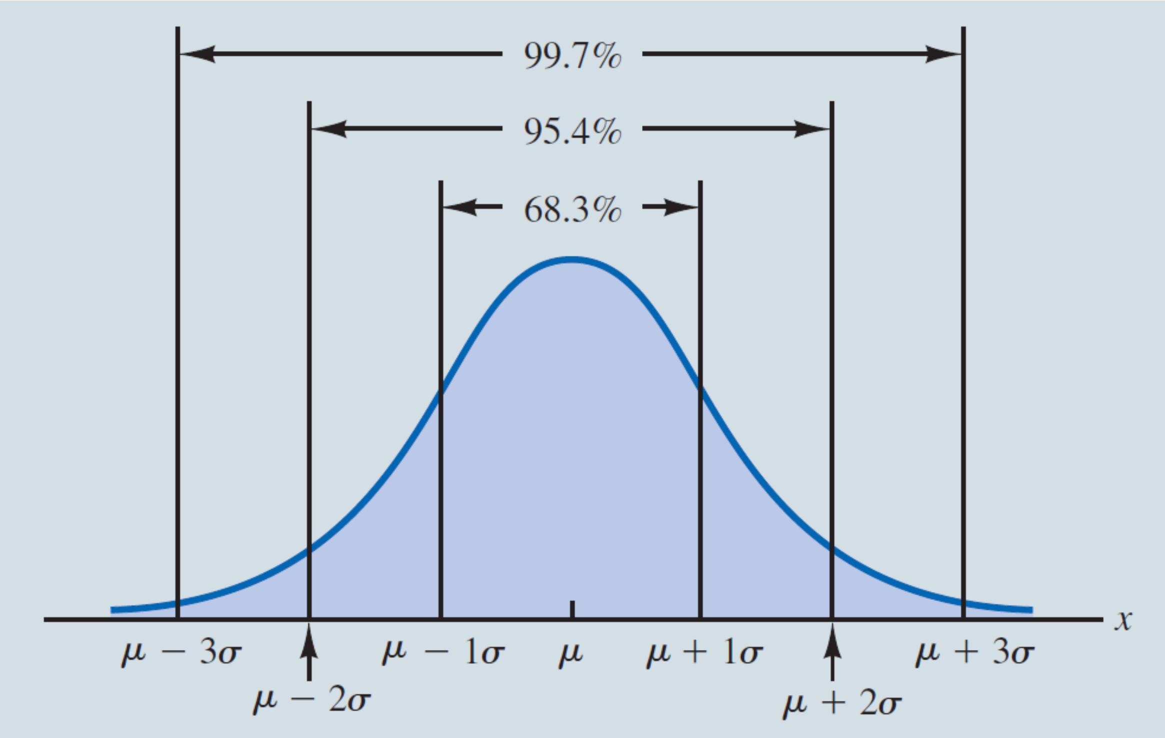


Normal Probability Distribution (9 of 10)

- Characteristics (basis for the empirical rule)
  - 68.3% of values of a normal random variable are within +/− 1 standard deviation of its mean.
  - 95.4% of values of a normal random variable are within +/− 2 standard deviations of its mean.
  - 99.7% of values of a normal random variable are within +/− 3 standard deviations of its mean.

Normal Probability Distribution (10 of 10)

- Characteristics (basis for the empirical rule)

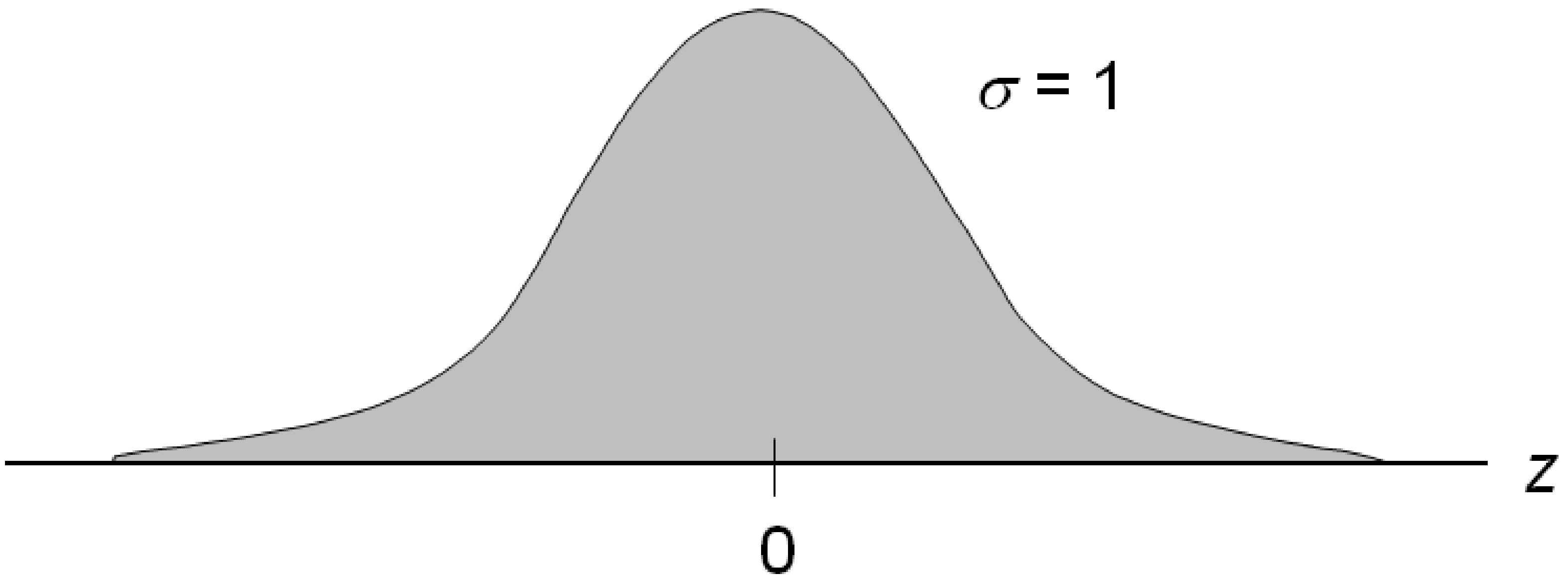


Standard Normal Probability Distribution (1 of 3)

- Characteristics
  - A random variable having a normal distribution with a mean of 0 and a standard deviation of 1 is said to have a standard normal probability distribution.

Standard Normal Probability Distribution (2 of 3)

- Characteristics
  - The letter *z* is used to designate the standard normal random variable.



Standard Normal Probability Distribution (3 of 3)

- Converting to Standard Normal Distribution

$$z = \frac{x - \mu}{\sigma}$$

- We can think of z as a measure of the number of standard deviations x is from μ.

Using Excel to Compute Standard Normal Probabilities (1 of 5)

- Excel has two functions for computing probabilities and z values for a standard normal probability distribution.
  - NORM.S.DIST function computes the cumulative probability given a z value.
  - NORM.S.INV function computes the z value given a cumulative probability.
    - “S” in the function names reminds us that these functions relate to the standard normal probability distribution.

Using Excel to Compute Standard Normal Probabilities (2 of 5)

- Excel Formula Worksheet

	A	B	C	D	E
1	**Probabilities: Standard Normal Distribution**				
2					
3			***P*(*z* <= 1)**	=NORM.S.DIST(1,TRUE)	
4			***P*(-.50 <= *z* <= 1.25)**	=NORM.S.DIST(1.25,TRUE)-NORM.S.DIST(-0.5,TRUE)	
5			***P*(-1.00 <= *z* <= 1.00)**	=NORM.S.DIST(1,TRUE)-NORM.S.DIST(-1,TRUE)	
6			***P*(*z* >= 1.58)**	=1-NORM.S.DIST(1.58,TRUE)	
7					
8					

Using Excel to Compute Standard Normal Probabilities (3 of 5)

- Excel Value Worksheet

	A	B	C	D	E
1	**Probabilities: Standard Normal Distribution**				
2					
3			***P*** **(*z* <= 1)**	0.8413	
4		***P*** **(-.50 <= *z* <= 1.25)**		0.5858	
5		***P*** **(-1.00 <= *z* <= 1.00)**		0.6827	
6			***P*** **(*z* >= 1.58)**	0.0571	
7					

Using Excel to Compute Standard Normal Probabilities (4 of 5)

- Excel Formula Worksheet

9		**Finding *z*-values Given Probabilities**	
10			
11		***z* value with .10 in upper tail**	=NORM.S.INV(0.9)
12		***z* value with .025 in upper tail**	=NORM.S.INV(0.975)
13		***z* value with .025 in lower tail**	=NORM.S.INV(0.025)
14			

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Using Excel to Compute Standard Normal Probabilities (5 of 5)

- Excel Value Worksheet

9	**Finding z-values Given Probabilities**	
10		
11	**z value with .10 in upper tail**	1.28
12	**z value with .025 in upper tail**	1.96
13	**z value with .025 in lower tail**	-1.96
14		

Standard Normal Probability Distribution (1 of 11)

- **Example:** Grear Tire Company Problem

  Grear Tire company has developed a new steel-belted radial tire to be sold through a chain of discount stores. But before finalizing the tire mileage guarantee policy, Grear's managers want probability information about the number of miles of tires will last.

Standard Normal Probability Distribution (2 of 11)

- **Example:** Grear Tire Company Problem

$$P(x > 40{,}000) = ?$$

It was estimated that the mean tire mileage is 36,500 miles with a standard deviation of 5000. The manager now wants to know the probability that the tire mileage x will exceed 40,000.

Standard Normal Probability Distribution (3 of 11)

- **Example:** Grear Tire Company Problem

  Solving for the Probability

  - Step 1: Convert *x* to standard normal distribution.

$$\begin{aligned} z &= (x - \mu)/\sigma \\ &= (40{,}000 - 36{,}500)/5{,}000 \\ &= .7 \end{aligned}$$

  - Step 2: Find the area under the standard normal curve to the left of $z = .7$.

Standard Normal Probability Distribution (4 of 11)

- **Example:** Grear Tire Company Problem

  Cumulative Probability Table for the Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.	.	.	.	.	.	.	.	.	.	.
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
.	.	.	.	.	.	.	.	.	.	.

$$P(z < .7) = .7580$$

Standard Normal Probability Distribution (5 of 11)

- **Example:** Grear Tire Company Problem

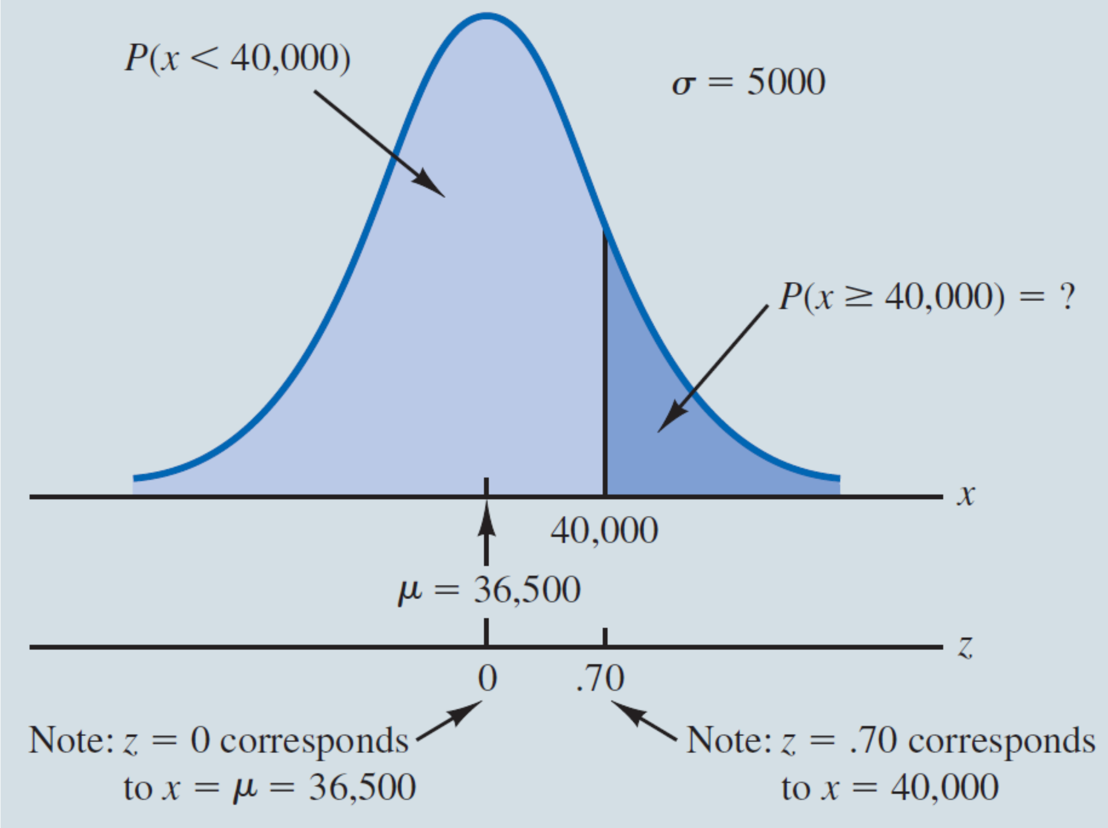
  Solving for the Probability

  - Step 3: Compute the area under the standard normal curve to the right of z = .7

$$
\begin{aligned}
P(z > .7) &= 1 - P(z < .7) \\
&= 1 - .7580 \\
&= .2420
\end{aligned}
$$

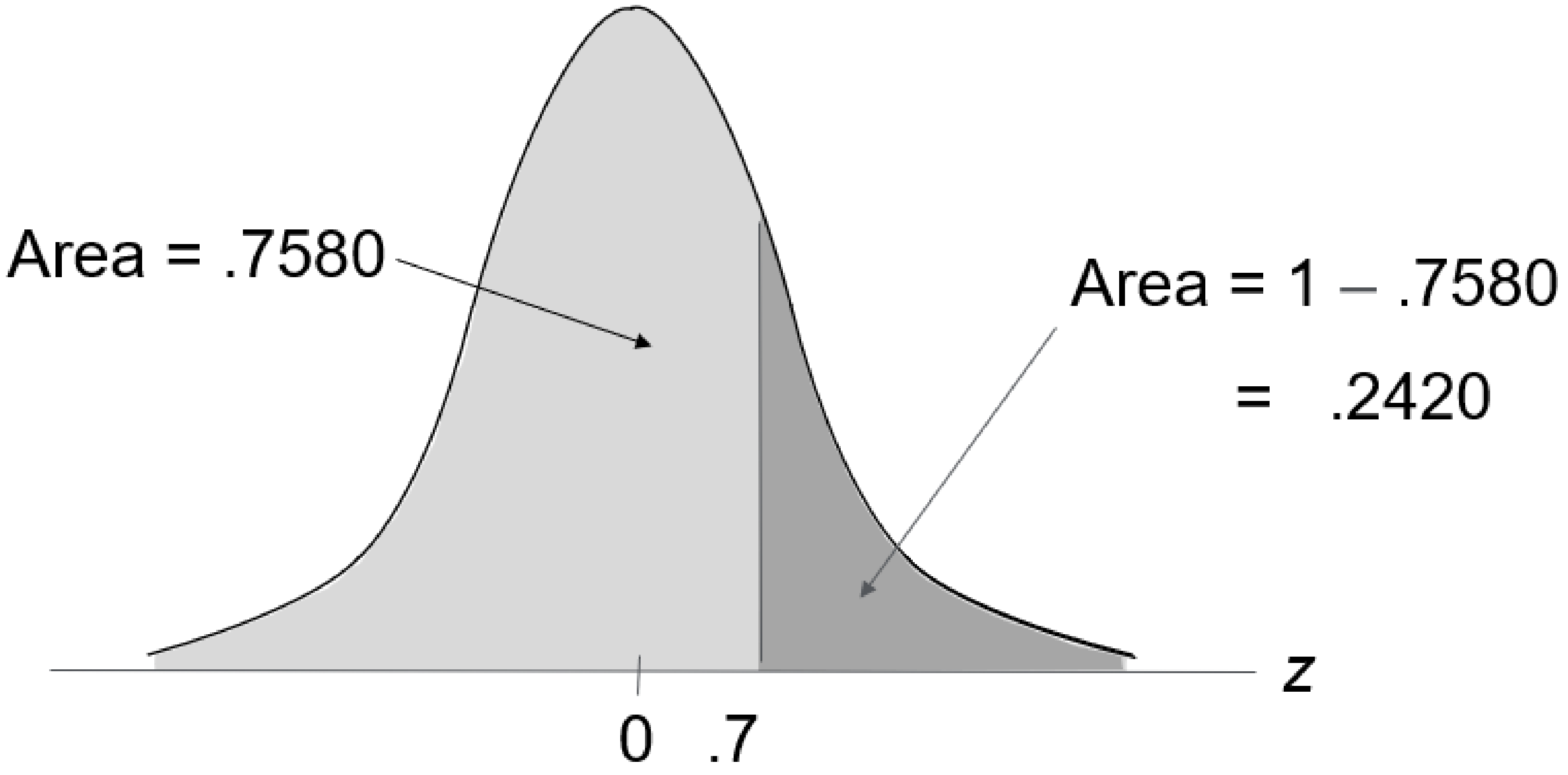
Standard Normal Probability Distribution (6 of 11)

- **Example:** Grear Tire Company Problem



Standard Normal Probability Distribution (7 of 11)

- **Example:** Grear Tire Company Problem

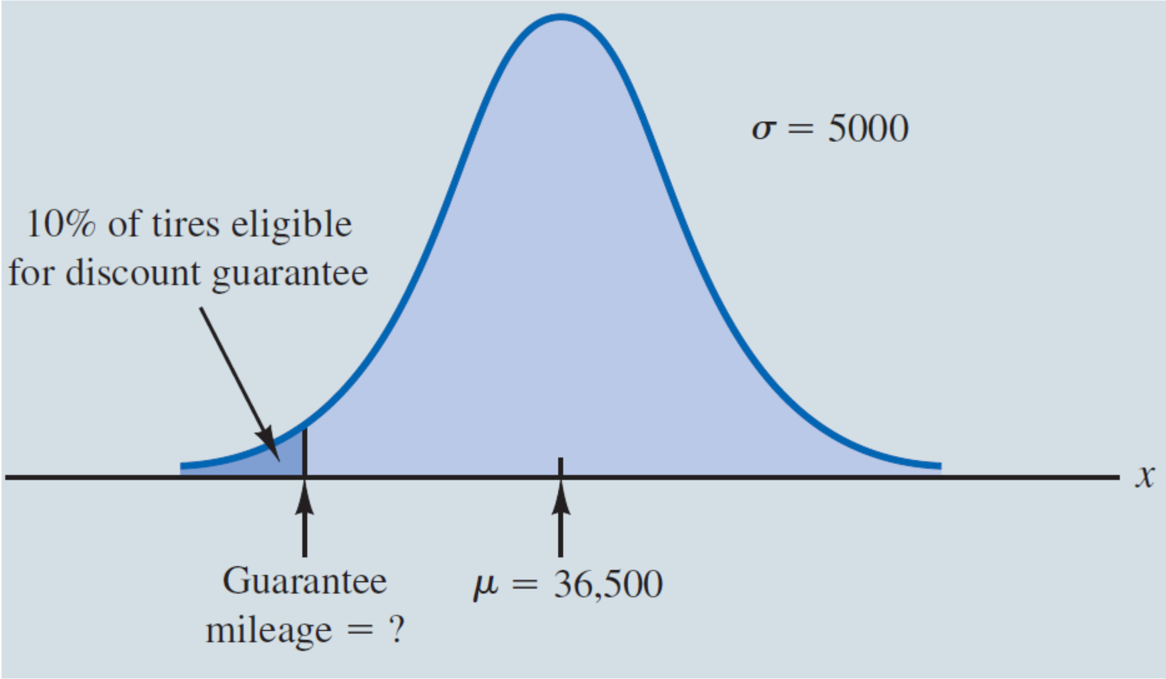


Standard Normal Probability Distribution (8 of 11)

- **Example:** Grear Tire Company Problem
  - What should be the guaranteed mileage if Grear wants no more than 10% of tires to be eligible for the discount guarantee?
  - (Hint: Given a probability, we can use the standard normal table in an inverse fashion to find the corresponding z value.)

Standard Normal Probability Distribution (9 of 11)

- **Example:** Grear Tire Company Problem
  - Solving for the guaranteed mileage



Standard Normal Probability Distribution (10 of 11)

- **Example:** Grear Tire Company Problem—Solving for the guaranteed mileage
  - Step 1: Find the z value that cuts off an area of .1 in the left tail of the standard normal distribution.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.	.	.	.	.	.	.	.	.	.	.
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
.	.	.	.	.	.	.	.	.	.	.

Standard Normal Probability Distribution (11 of 11)

- From the table we see that $z = -1.28$ cuts off an area of 0.1 in the lower tail.
  - Step 2: Convert $z_{.1}$ to the corresponding value of x.

$$x = \mu + z_{.1}\sigma$$

$$x = 36{,}500 - 1.28(5000) = 30{,}100$$

- Thus a guarantee of 30,100 miles will meet the requirement that approximately 10% of the tires will be eligible for the guarantee.

Using Excel to Compute Normal Probabilities (1 of 3)

- Excel has two functions for computing cumulative probabilities and *x* values for any normal distribution:
  - NORM.DIST is used to compute the cumulative probability given an *x* value.
  - NORM.INV is used to compute the x value given a cumulative probability.

Using Excel to Compute Normal Probabilities (2 of 3)

- Excel Formula Worksheet

	A	B	C	D
1			**Probabilities: Normal Distribution**	
2				
3			***P*(*x* <= 20000)**	=NORM.DIST(20000,36500,5000,TRUE)
4			***P*(20000 <= *x* <= 40000)**	=NORMDIST(40000,36500,5000,TRUE)-NORM.DIST(20000,36500,5000,TRUE)
5			***P*(*x* >= 40000)**	=1-NORM.DIST(40000,36500,5000,TRUE)
6				
7			**Finding *x* values Given Probabilities**	
8				
9			***x* value with .10 in lower tail**	=NORM.INV(0.1,36500,5000)
10			***x* value with .025 in upper tail**	=NORM.INV(0.975,36500,5000)
11				

Using Excel to Compute Normal Probabilities (3 of 3)

- Excel Value Worksheet

	A	B	C	D	E
1	**Probabilities: Normal Distribution**				
2					
3			**$P(x <= 20000)$**	0.0005	
4			**$P(20000 <= x <= 40000)$**	0.7576	
5			**$P(x >= 40000)$**	0.2420	
6					
7	**Finding x values Given Probabilities**				
8					
9			**x value with .10 in lower tail**	30092.24	
10			**x value with .025 in upper tail**	46299.82	
11					

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Exponential Probability Distribution (1 of 6)

- The exponential probability distribution is useful in describing the time it takes to complete a task.
- The exponential random variables can be used to describe:
  - Time between vehicle arrivals at a toll booth
  - Time required to complete a questionnaire
  - Distance between major defects in a highway
- In waiting line applications, the exponential distribution is often used for service times.

Exponential Probability Distribution (2 of 6)

- A property of the exponential distribution is that the mean and standard deviation are equal.
- The exponential distribution is skewed to the right. Its skewness measure is 2.

Exponential Probability Distribution (3 of 6)

- Density Function

$$f(x) = \frac{1}{\mu} e^{-x/\mu} \text{ for } x \geq 0$$

where: μ = expected value or mean

$e = 2.71828$

Exponential Probability Distribution (4 of 6)

- Cumulative Probabilities

$$P(x \le x_0) = 1 - e^{-x_0/\mu}$$

where:

x_0 = some specific value of x

Exponential Probability Distribution (5 of 6)

- **Example:** Loading time for trucks

  Suppose x represents the loading time for a truck at the Schips loading dock and follows exponential distribution. If the mean or average loading time is 15 minutes, what is the probability that loading a truck will take 6 minutes or less?

Exponential Probability Distribution (6 of 6)

- **Example:** Loading time for trucks

$$P(x \le x_0) = 1 - e^{-x_0/\mu}$$

$$P(x \le 6) = 1 - e^{-\frac{6}{15}}$$

$$= .3297$$

Using Excel to Compute Exponential Probabilities (1 of 4)

- The EXPON.DIST function can be used to compute exponential probabilities.
- The EXPON.DIST function has three inputs:
  - 1st The value of the random variable *x*
  - 2nd $1/\mu$: the inverse of the mean number of occurrences in an interval
  - 3rd "TRUE" or "FALSE: We will enter "TRUE" if a cumulative probability is desired and "FALSE" if the height of the probability function is desired. We will always enter TRUE because we will be computing cumulative probabilities.

Using Excel to Compute Exponential Probabilities (2 of 4)

- Excel Formula Worksheet

	A	B	C	D
1			**Probabilities: Exponential Distribution**	
2				
3			***P*(*x* <= 18)**	=EXPON.DIST(18,1/15,TRUE)
4			***P(6 <= x <= 18)***	=EXPON.DIST(18,1/15,TRUE)-EXPON.DIST(6,1/15,TRUE)
5			***P*(*x* >= 8)**	=1-EXPON.DIST(8,1/15,TRUE)
6				

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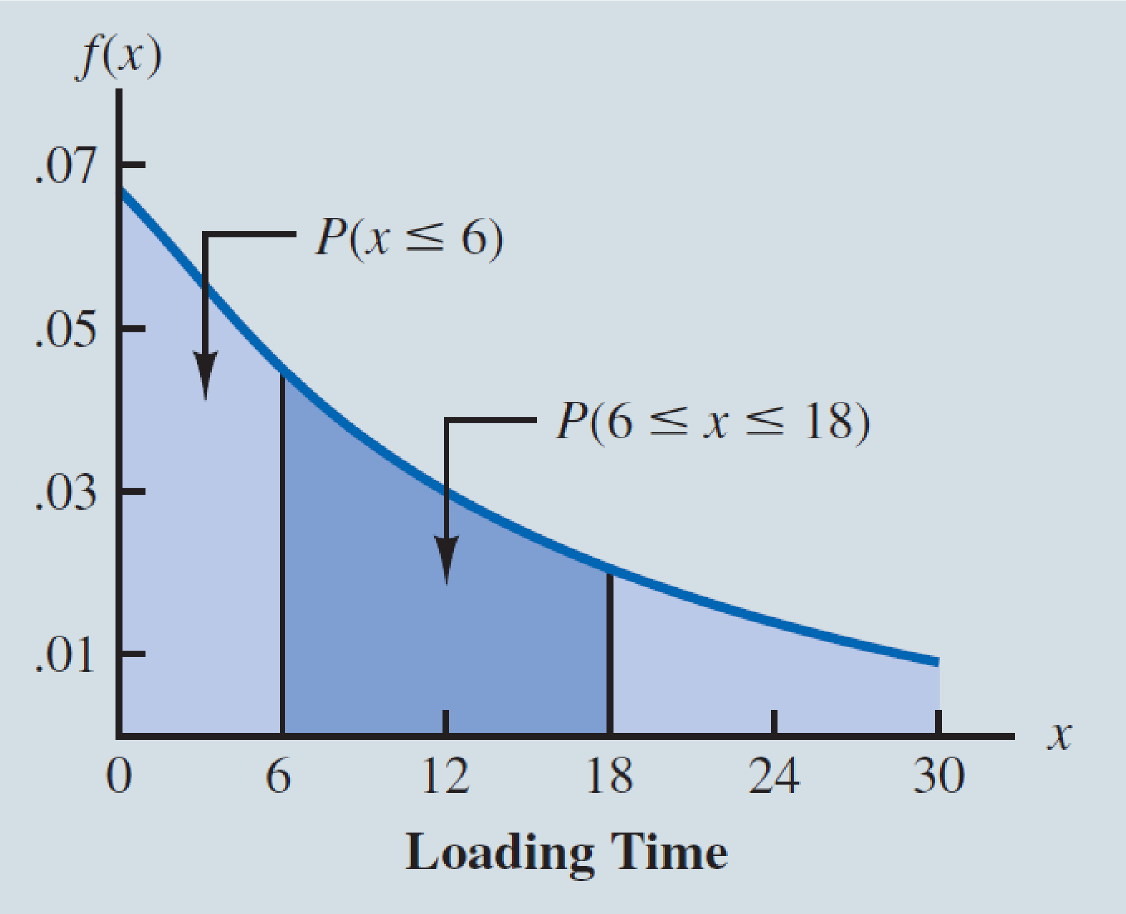
Using Excel to Compute Exponential Probabilities (3 of 4)

- Excel Value Worksheet

	A	B	C	D	E
1	**Probabilities: Exponential Distribution**				
2					
3			$P(x <= 18)$	0.6988	
4			$P(6 <= x <= 18)$	0.3691	
5			$P(x >= 8)$	0.5866	
6					

Using Excel to Compute Exponential Probabilities (4 of 4)



Relationship between the Poisson and Exponential Distributions