

Chapter 9

Hypothesis Testing

- Developing Null and Alternative Hypotheses
- Type I and Type II Errors
- Population Mean: σ Known
- Population Mean: σ Unknown
- Population Proportion

Hypothesis Testing

- Hypothesis testing can be used to determine whether a statement about the value of a population parameter should or should not be rejected.
- The null hypothesis, denoted by H_0 , is a tentative assumption about a population parameter.
- The alternative hypothesis, denoted by H_a , is the opposite of what is stated in the null hypothesis.
- The hypothesis testing procedure uses data from a sample to test the two competing statements indicated by H_0 and H_a .

Developing Null and Alternative Hypotheses (1 of 7)

- It is not always obvious how the null and alternative hypotheses should be formulated.
- Care must be taken to structure the hypotheses appropriately so that the test conclusion provides the information the researcher wants.
- The context of the situation is very important in determining how the hypotheses should be stated.
- In some cases it is easier to identify the alternative hypothesis first. In other cases the null is easier.
- Correct hypothesis formulation will take practice.

Developing Null and Alternative Hypotheses (2 of 7)

- Alternative Hypothesis as a Research Hypothesis
 - Many applications of hypothesis testing involve an attempt to gather evidence in support of a research hypothesis.
 - In such cases, it is often best to begin with the alternative hypothesis and make it the conclusion that the researcher hopes to support.
 - The conclusion that the research hypothesis is true is made if the sample data provide sufficient evidence to show that the null hypothesis can be rejected.

Developing Null and Alternative Hypotheses (3 of 7)

- Alternative Hypothesis as a Research Hypothesis

Example:

A new teaching method is developed that is believed to be better than the current method.

- Alternative Hypothesis:

The new teaching method is better.

- Null Hypothesis:

The new method is no better than the old method.

Developing Null and Alternative Hypotheses (4 of 7)

- Alternative Hypothesis as a Research Hypothesis

Example:

A new sales force bonus plan is developed in an attempt to increase sales.

- Alternative Hypothesis:

The new bonus plan increases sales.

- Null Hypothesis:

The new bonus plan does not increase sales.

Developing Null and Alternative Hypotheses (5 of 7)

- Alternative Hypothesis as a Research Hypothesis

Example:

A new drug is developed with the goal of lowering blood pressure more than the existing drug.

- Alternative Hypothesis:

The new drug lowers blood pressure more than the existing drug.

- Null Hypothesis:

The new drug does not lower blood pressure more than the existing drug.

Developing Null and Alternative Hypotheses (6 of 7)

The null hypothesis as an assumption to be challenged:

- We might begin with a belief or assumption that a statement about the value of a population parameter is true.
- We then using a hypothesis test to challenge the assumption and determine if there is statistical evidence to conclude that the assumption is incorrect.
- In these situations, it is helpful to develop the null hypothesis first.

Developing Null and Alternative Hypotheses (7 of 7)

The null hypothesis as an assumption to be challenged:

Example:

The label on a soft drink bottle states that it contains 67.6 fluid ounces.

- Null Hypothesis:

The label is correct. $\mu \geq 67.6$ ounces.

- Alternative Hypothesis:

The label is incorrect. $\mu < 67.6$ ounces.

Summary of Forms for Null and Alternative Hypotheses about a Population Mean

The equality part of the hypotheses always appears in the null hypothesis.

In general, a hypothesis test about the value of a population mean μ must take one of the following three forms (where μ_0 is the hypothesized value of the population mean).

One-tailed (lower-tail): $H_0: \mu \geq \mu_0$ $H_a: \mu < \mu_0$

One-tailed (upper-tail): $H_0: \mu \leq \mu_0$ $H_a: \mu > \mu_0$

Two-tailed test: $H_0: \mu = \mu_0$ $H_a: \mu \neq \mu_0$

Null and Alternative Hypotheses (1 of 2)

Example: Metro EMS

A major West Coast city hospital provides one of the most comprehensive emergency medical services in the world. Operating in a multiple hospital system with approximately 20 mobile medical units, the service goal is to respond to medical emergencies with a mean time of 12 minutes or less.

The director of medical services wants to formulate a hypothesis test that could use a sample of emergency response times to determine whether or not the service goal of 12 minutes or less is being achieved.

Null and Alternative Hypotheses (2 of 2)

$H_0 : \mu \leq 12$ The emergency service is meeting the response goal; no follow-up action is necessary.

$H_a : \mu > 12$ The emergency service is not meeting the response goal; appropriate follow-up action is necessary.

where: μ = mean response time for the population of medical emergency requests

Type I Error

- Because hypothesis tests are based on sample data, we must allow for the possibility of errors.
- A Type I error is rejecting H_0 when it is true.
- The probability of making a Type I error when the null hypothesis is true as an equality is called the level of significance.
- Applications of hypothesis testing that only control the Type I error are often called significance tests.

Type II Error

- A Type II error is accepting H_0 when it is false.
- It is difficult to control the probability of making a Type II error.
- Statisticians avoid the risk of making a Type II error by using “do not reject H_0 ” and not “accept H_0 ”.

Type I and Type II Errors

Example: Metro EMS

		Population Condition	
		H_0 True ($\mu \leq 12$)	H_0 False ($\mu > 12$)
Conclusion			
Accept H_0 (Conclude $\mu \leq 12$)	Correct Conclusion	Type II Error	
Reject H_0 (Conclude $\mu > 12$)	Type I Error	Correct Conclusion	

p -Value Approach to One-Tailed Hypothesis Testing

- The p -value is the probability computed using the test statistic, that measures the support (or lack of support) provided by the sample for the null hypothesis.
- If the p -value is less than or equal to the level of significance α , the value of the test statistic is in the rejection region.
- Reject H_0 if the p -value $\leq \alpha$.

Suggested Guidelines for Interpreting p -Values

- Less than .01

Overwhelming evidence to conclude H_a is true.

- Between .01 and .05

Strong evidence to conclude H_a is true.

- Between .05 and .10

Weak evidence to conclude H_a is true.

- Greater than .10

Insufficient evidence to conclude H_a is true.

Critical Value Approach to One-Tailed Hypothesis Testing (1 of 2)

- The test statistic z has a standard normal probability distribution.
- We can use the standard normal probability distribution table to find the z -value with an area of α in the lower (or upper) tail of the distribution.
- The value of the test statistic that establishes the boundary of the rejection region is called the critical value for the test.
- The rejection rule is:

Lower tail: Reject H_0 if $z \leq -z_\alpha$

Upper tail: Reject H_0 if $z \geq z_\alpha$

Steps of Hypothesis Testing (1 of 2)

- Step 1. Develop the null and alternative hypotheses.
- Step 2. Specify the level of significance α .
- Step 3. Collect the sample data and compute the value of the test statistic.

p -Value Approach

- Step 4. Use the value of the test statistic to compute the p -value.
- Step 5. Reject H_0 if $p\text{-value} \leq \alpha$.

Steps of Hypothesis Testing (2 of 2)

Critical Value Approach

- Step 4. Use the level of significance α to determine the critical value and the rejection rule.
- Step 5. Use the value of the test statistic and the rejection rule to determine whether to reject H_0 .

Lower-Tailed Test about a Population Mean: σ Known (1 of 2)

Example: Hilltop Coffee

FTC wants to check Hilltop's claim that its large can of coffee contains 3 pounds of coffee. The FTC director wants to perform a hypothesis test, with a .01 level of significance, to check Hilltop's claim. Suppose a random sample of 36 cans provides a sample mean of $\bar{x} = 2.92$ pounds.

Previous FTC tests show that the population standard deviation can be assumed known with a value of $\sigma = 0.18$ and also show that the population of filling weights can be assumed to have a normal distribution.

One-Tailed Tests about a Population Mean: σ Known (1 of 3)

Example: Hilltop Coffee

p -Value and Critical Value Approaches

1. Develop the hypotheses.

$$H_0 : \mu \geq 3$$

$$H_a : \mu < 3$$

2. Specify the level of significance.

$$\alpha = .01$$

3. Compute the value of the test statistic.

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{2.92 - 3}{.18/\sqrt{36}} = -2.67$$

One-Tailed Tests about a Population Mean: σ Known (2 of 3)

Example: Hilltop Coffee

p -value Approach

4. Compute the p -value.

For $z = -2.67$, using standard normal probability, we find that the lower tail area is .0038.

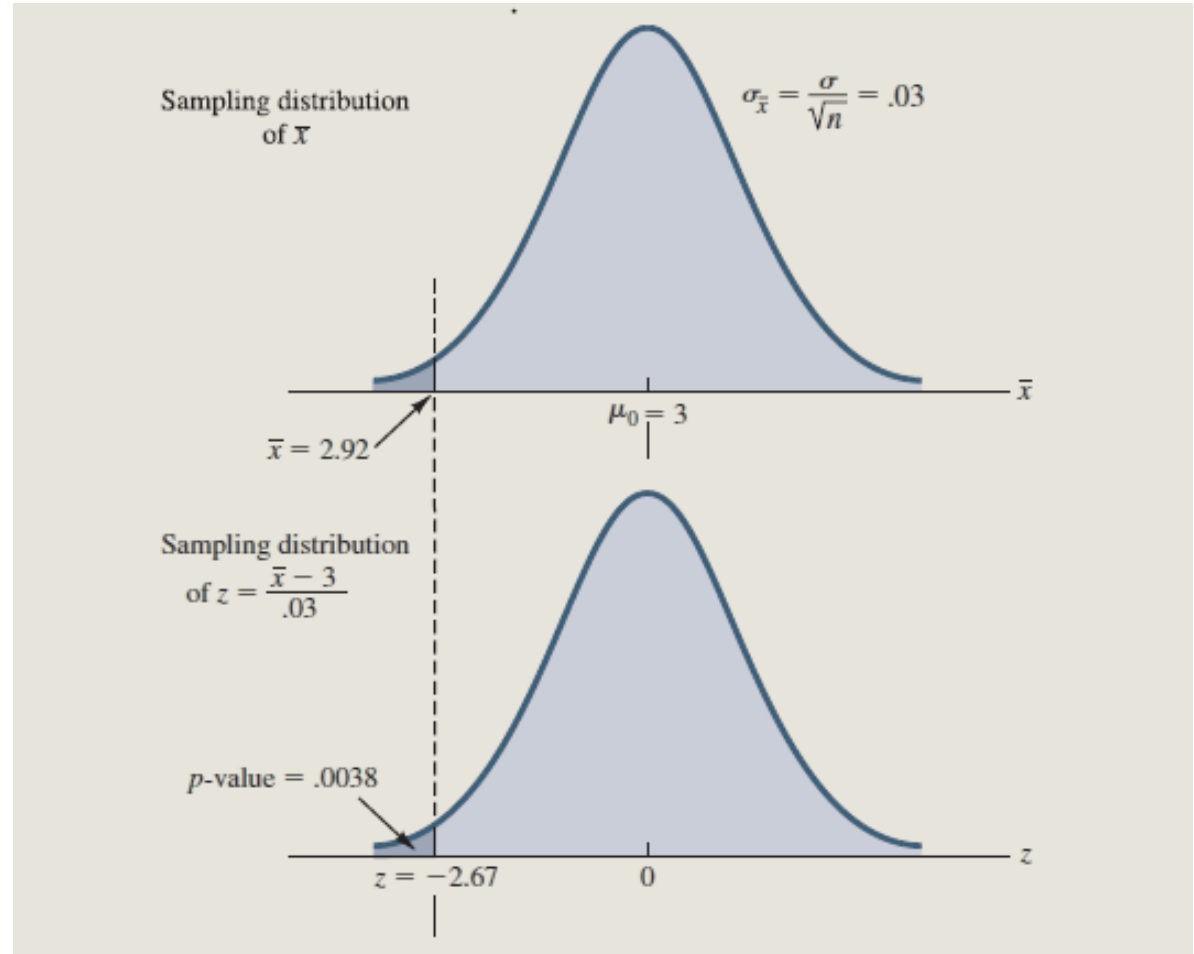
5. Determine whether to reject H_0 .

Because $p\text{-value } .0038 \leq \alpha = .01$, we reject H_0 .

Lower-Tailed Test about a Population Mean: σ Known (2 of 2)

Example: Hilltop Coffee

p-value: $\leq \alpha$ so reject H_0 .



One-Tailed Tests about a Population Mean: σ Known (3 of 3)

Example: Hilltop Coffee

Critical Value Approach

4. Determine the critical value and rejection rule.

For $\alpha = .01$, $z = -2.33$

Reject H_0 if $z \leq -2.33$

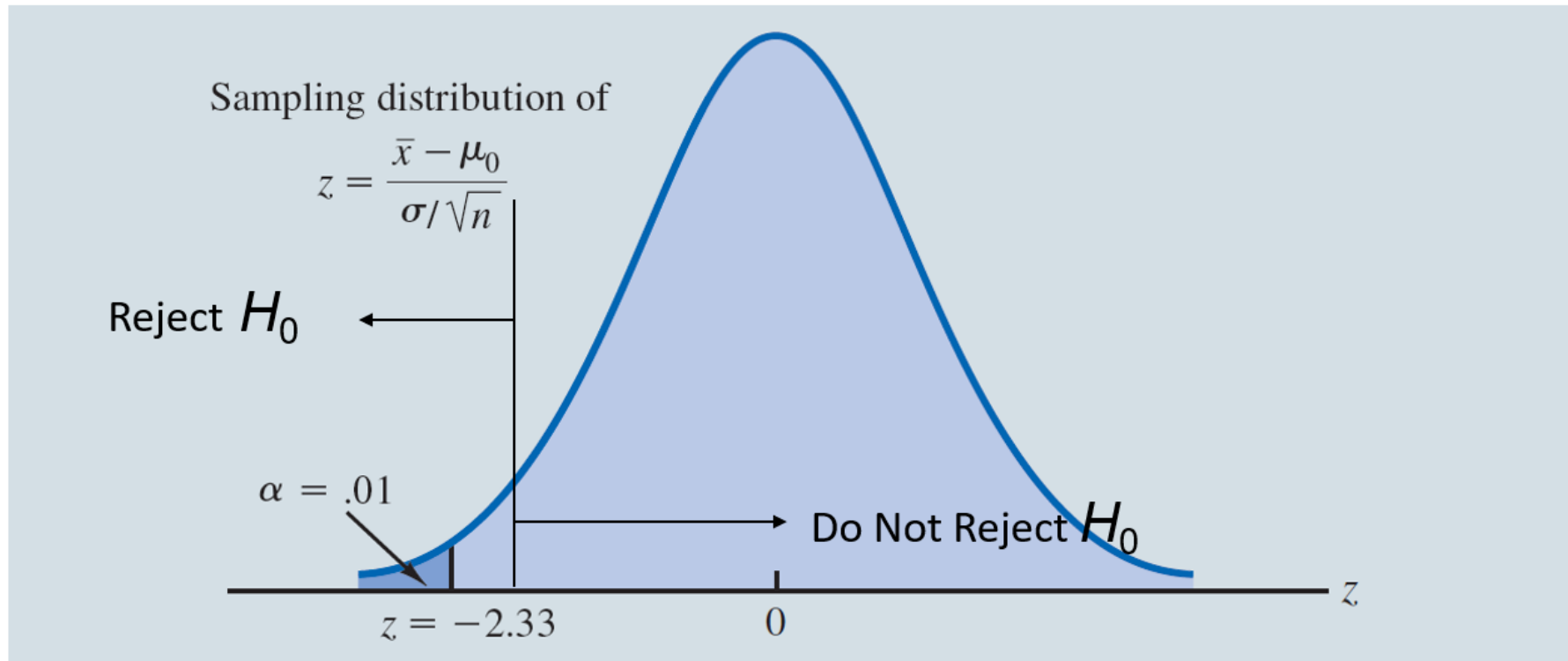
5. Determine whether to reject H_0 .

Because $-2.67 < -2.33$, we reject H_0 .

We conclude that Hilltop Coffee is underfilling cans.

Critical Value Approach to One-Tailed Hypothesis Testing (2 of 2)

Example: Hilltop Coffee



p -Value Approach to Two-Tailed Hypothesis Testing

Compute the p -value using the following three steps:

1. Compute the value of the test statistic z .
2. If z is in the upper tail ($z > 0$), compute the probability that z is greater than or equal to the value of the test statistic. If z is in the lower tail ($z < 0$), compute the probability that z is less than or equal to the value of the test statistic.
3. Double the tail area obtained in step 2 to obtain the p -value.

The rejection rule: Reject H_0 if the p -value $\leq \alpha$.

Critical Value Approach to Two-Tailed Hypothesis Testing

- The critical values will occur in both the lower and upper tails of the standard normal curve.
- Use the standard normal probability distribution table to find $z_{\alpha/2}$ (the z-value with an area of $\alpha/2$ in the upper tail of the distribution).
- The rejection rule is: Reject H_0 if $z \leq -z_{\alpha/2}$ or $z \geq z_{\alpha/2}$.

Two-Tailed Tests about a Population Mean: σ Known (1 of 9)

Example: MaxFlight Inc.

MaxFlight produces golf balls with a mean driving distance of 295 yards. Its quality control program involves taking periodic samples of 50 golf balls to monitor the manufacturing process.

Quality assurance procedures call for the continuation of the process if the sample results are consistent with the assumption that the mean driving distance for the population of golf balls is 295 yards; otherwise the process will be adjusted.

Two-Tailed Tests about a Population Mean: σ Known (2 of 9)

Example: MaxFlight Inc.

Assume that a sample of 50 golf balls provided a sample mean of 297.6 yards. The population standard deviation is believed to be 12 yards.

Perform a hypothesis test, at the .05 level of significance, to help determine whether the ball manufacturing process should continue operating or be stopped and corrected.

Two-Tailed Tests about a Population Mean: σ Known (3 of 9)

Example: MaxFlight Inc.

p-value and Critical Value Approaches

1. Determine the hypotheses.
 $H_0 : \mu = 295$
 $H_a : \mu \neq 295$

2. Specify the level of significance.
 $\alpha = .05$

3. Compute the value of the test statistic.

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{297.6 - 295}{12/\sqrt{50}} = 1.53$$

Two-Tailed Tests about a Population Mean: σ Known (4 of 9)

Example: MaxFlight Inc.

p-value Approach

4. Compute the *p*-value.

For $z = 1.53$, the standard normal distribution table shows $P(z < 1.53) = .9370$

$$p\text{-value} = 2(1 - .9370) = .1260$$

5. Determine whether to reject H_0 .

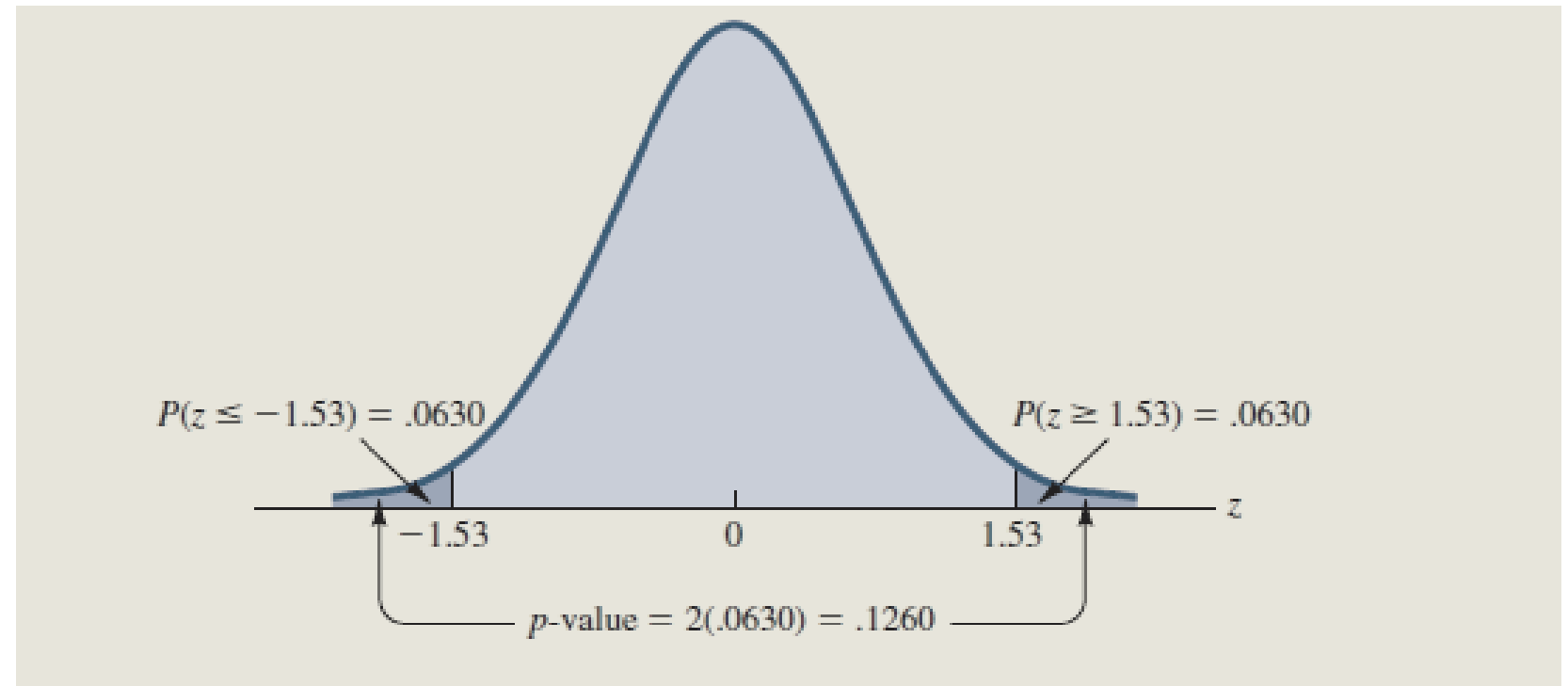
Because $p\text{-value} = .1260 > \alpha = .05$, we do not reject H_0 .

There is sufficient statistical evidence to infer that the null hypothesis is true and so no action needs to be taken to adjust MaxFlight's manufacturing process.

Two-Tailed Tests about a Population Mean: σ Known (5 of 9)

Example: MaxFlight Inc.

p-Value Approach



Two-Tailed Tests about a Population Mean: σ Known (6 of 9)

Example: MaxFlight Inc.

Critical Value Approach

4. Determine the critical value and rejection rule.

$$\text{For } \alpha/2 = .05/2 = .025, z_{.025} = 1.96$$

$$\text{Reject } H_0 \text{ if } z \leq -1.96 \text{ or } z \geq 1.96$$

5. Determine whether to reject H_0 .

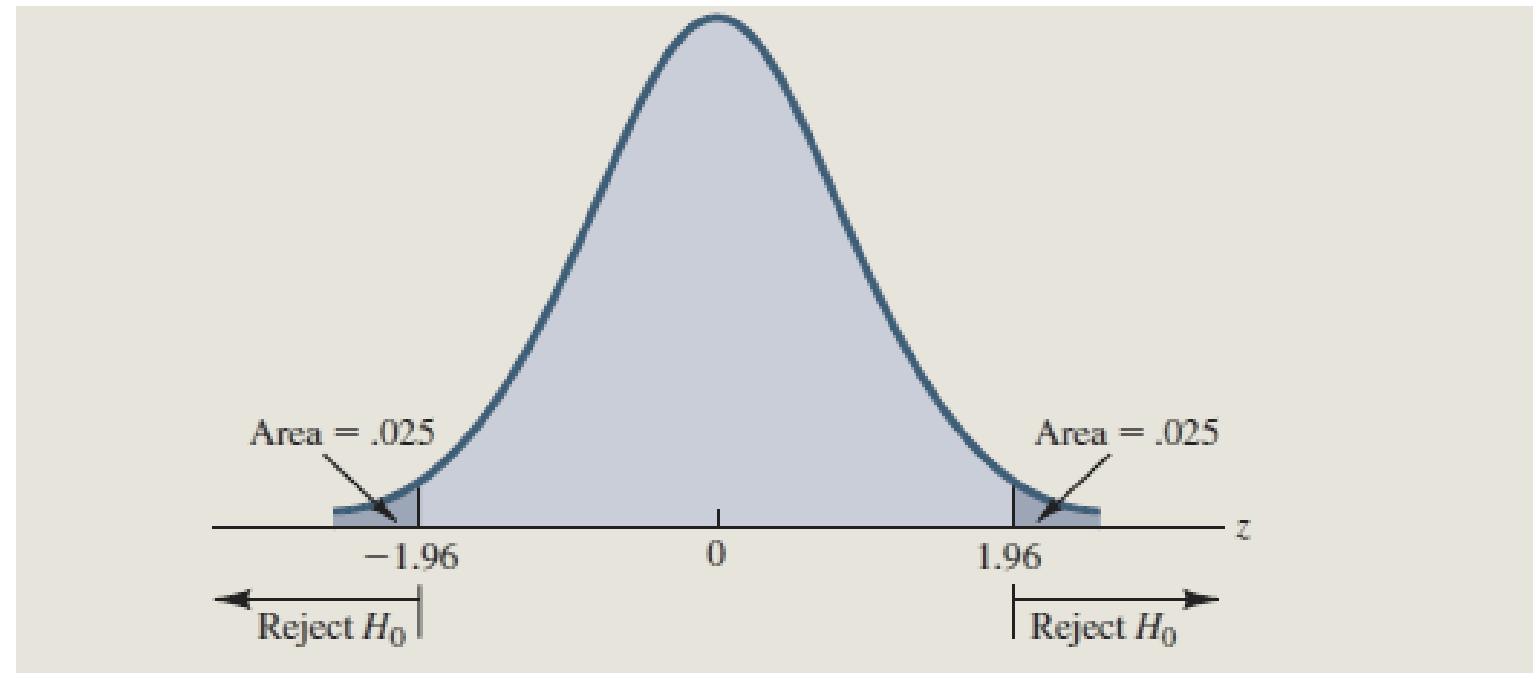
Because $1.53 \leq 1.96$, we cannot reject H_0 .

There is sufficient statistical evidence to infer that the null hypothesis cannot be rejected.

Two-Tailed Tests about a Population Mean: σ Known (7 of 9)

Example: MaxFlight Inc.

Critical Value Approach



Two-Tailed Tests about a Population Mean: σ Known (8 of 9)

Excel Formula Worksheet

Note: Rows 16 to 49 are not shown.

	A	B	C	D
1	Yards		Hypothesis Test about a Population Mean: σ Known Case	
2	303			
3	282			
4	289		Sample Size	=COUNT(A2:A51)
5	298		Sample Mean	=AVERAGE(A2:A51)
6	283			
7	317		Population Standard Deviation	12
8	297		Hypothesized Value	295
9	308			
10	317		Standard Error	=D7/SQRT(D4)
11	293		Test Statistic z	=(D5-D8)/D10
12	284			
13	290		<i>p</i> -value (Lower Tail)	=NORM.S.DIST(D11,TRUE)
14	304		<i>p</i> -value (Upper Tail)	=1-D13
15	290		<i>p</i> -value (Two Tail)	=2*(MIN(D13,D14))
16	311			
50	301			
51	292			

Two-Tailed Tests About a Population Mean: σ Known (9 of 9)

Excel Value Worksheet

Note: Rows 16 to 49 are not shown.

	A	B	C	D	E
1	Yards		Hypothesis Test about a Population Mean: σ Known Case		
2	303				
3	282				
4	289		Sample Size	50	
5	298		Sample Mean	297.6	
6	283				
7	317		Population Standard Deviation	12	
8	297		Hypothesized Value	295	
9	308				
10	317		Standard Error	1.6971	
11	293		Test Statistic z	1.5321	
12	284				
13	290		<i>p</i> -value (Lower Tail)	0.9372	
14	304		<i>p</i> -value (Upper Tail)	0.0628	
15	290		<i>p</i> -value (Two Tail)	0.1255	
16	311				
50	301				
51	292				
52					

Relationship between Interval Estimation and Hypothesis Testing (1 of 2)

- Select a simple random sample from the population and use the value of the sample mean \bar{x} to develop the confidence interval for the population mean μ .
(Confidence intervals are covered in Chapter 8.)
- If the confidence interval contains the hypothesized value μ_0 , do not reject H_0 . Otherwise, reject H_0 . (Actually, H_0 should be rejected if μ_0 happens to be equal to one of the end points of the confidence interval.)

Relationship between Interval Estimation and Hypothesis Testing (2 of 2)

Example: MaxFlight Inc.

- The 97% confidence interval for μ is

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 297.6 \pm 1.96 \left(12 / \sqrt{50} \right) = 297.6 \pm 3.3$$

294.3 to 300.9

- Because the hypothesized value for the population mean, $\mu_0 = 295$, is in this, the null hypothesis, $H_0 : \mu = 295$, cannot be rejected.

Tests About a Population Mean: σ Unknown (1 of 2)

Test Statistic:

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

This test statistic has a t distribution with $n - 1$ degrees of freedom.

Tests about a Population Mean: σ Unknown (2 of 2)

- Rejection Rule: p -value Approach

Reject H_0 if $p\text{-value} \leq \alpha$

- Rejection Rule: Critical Value Approach

$H_0 : \mu \geq \mu_0$ Reject H_0 if $t \leq -t_\alpha$

$H_0 : \mu \leq \mu_0$ Reject H_0 if $t \geq t_\alpha$

$H_0 : \mu = \mu_0$ Reject H_0 if $t \leq -t_{\alpha/2}$ or $t \geq t_{\alpha/2}$

p -Values and the t Distribution (1 of 2)

- The format of the t distribution table provided in most statistics textbooks does not have sufficient detail to determine the exact p -value for a hypothesis test.
- However, we can still use the t distribution table to identify a range for the p -value.
- An advantage of computer software packages is that the computer output will provide the p -value for the t distribution.

p -Values and the t Distribution (2 of 2)

Example:

A business travel magazine wants to classify transatlantic gateway airports according to the mean rating for the population of business travelers. A rating scale with a low score of 0 and a high score of 10 will be used, and airports with a population mean rating greater than 7 will be designated as superior service airports.

A sample of 60 business travelers were surveyed at London's Heathrow Airport, which provided a sample mean rating of $\bar{x} = 7.25$ and a sample standard deviation of $s = 1.052$.

Do the data indicate that Heathrow should be designated as a superior service airport?

One-Tailed Test about a Population Mean: σ Unknown (1 of 3)

p-value and Critical Value Approaches

1. Determine the hypotheses.

$$H_0 : \mu \leq 7$$

$$H_a : \mu > 7$$

2. Specify the level of significance.

$$\alpha = .05$$

3. Compute the value of the test statistic.

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{7.25 - 7}{1.052/\sqrt{60}} = 1.84$$

One-Tailed Test about a Population Mean: σ Unknown (2 of 3)

p -value Approach

4. Compute the p -value. For $t = 1.84$, the p -value must be less than .05 (for $t = 1.671$) and greater than 0.25 (for $t = 2.001$).

$$.025 < p\text{-value} < \alpha = .05,$$

5. Determine whether to reject H_0 .

Because $p\text{-value} \leq \alpha = .05$, we reject H_0 .

We reject the null hypothesis. Heathrow should be classified as a superior service airport.

One-Tailed Test about a Population Mean: σ Unknown (3 of 3)

Critical Value Approach

- Determine the critical value and rejection rule.

For $\alpha = .05$ and d.f. = $60 - 1 = 59$, $t_{.05} = 1.671$

Reject H_0 if $t \geq 1.671$

- Determine whether to reject H_0 .

Because $1.84 > 1.669$, we reject H_0 . Heathrow should be classified as a superior service airport.

Using Excel: Hypothesis Test for the σ Unknown Case

Excel Formula and Value Worksheet

	A	B	C	D	E
1	Units		Hypothesis Test about a Population Mean: σ Unknown Case		
2	26				
3	23				
4	32		Sample Size	=COUNT(A2:A26)	
5	47		Sample Mean	=AVERAGE(A2:A26)	
6	45		Sample Standard Deviation	=STDEV.S(A2:A26)	
7	31				
8	47		Hypothesized Value	40	
9	59				
10	21		Standard Error	=D6/SQRT(D4)	
11	52		Test Statistic t	=(D5-D8)/D10	
12	45		Degrees of Freedom	=D4-1	
13	53				
14	34		p -value (Lower Tail)	=T.DIST(D11,D12,TRUE)	
15	45		p -value (Upper Tail)	=1-D14	
16	39		p -value (Two Tail)	=2*MIN(D14,D15)	
17	52				
25	30				
26	28				
27					

	A	B	C	D	E	F
1	Units		Hypothesis Test about a Population Mean: σ Unknown Case			
2	26					
3	23					
4	32		Sample Size	25		
5	47		Sample Mean	37.4		
6	45		Sample Standard Deviation	11.79		
7	31					
8	47		Hypothesized Value	40		
9	59					
10	21		Standard Error	2.3580		
11	52		Test Statistic t	-1.1026		
12	45		Degrees of Freedom	24		
13	53					
14	34		p -value (Lower Tail)	0.1406		
15	45		p -value (Upper Tail)	0.8594		
16	39		p -value (Two Tail)	0.2811		
17	52					
25	30					
26	28					
27						

A Summary of Forms for Null and Alternative Hypotheses about a Population Proportion

- The equality part of the hypotheses always appears in the null hypothesis.
- In general, a hypothesis test about the value of a population proportion p must take one of the following three forms (where p_0 is the hypothesized value of the population proportion).

One-tailed (lower tail) $H_0: p \geq p_0$ $H_a: p < p_0$

One-tailed (upper tail) $H_0: p \leq p_0$ $H_a: p > p_0$

Two-tailed $H_0: p = p_0$ $H_a: p \neq p_0$

Tests about a Population Proportion (1 of 6)

Test Statistic:

$$z = \frac{\bar{p} - p_0}{\sigma_{\bar{p}}}$$

where:

$$\sigma_{\bar{p}} = \sqrt{\frac{p_0(1-p_0)}{n}}$$

assuming $np \geq 5$ and $n(1-p) \geq 5$

Tests about a Population Proportion (2 of 6)

- Rejection Rule: p -value Approach

$$H_0 \text{ if } p\text{-value} \leq \alpha$$

- Rejection Rule: Critical Value Approach

$$H_0 : p \leq p_0 \quad \text{Reject } H_0 \text{ if } z \geq z_\alpha$$

$$H_0 : p \geq p_0 \quad \text{Reject } H_0 \text{ if } z \leq -z_\alpha$$

$$H_0 : p = p_0 \quad \text{Reject } H_0 \text{ if } z \leq -z_{\alpha/2} \text{ or } z \geq z_{\alpha/2}$$

Tests about a Population Proportion (3 of 6)

Example: Pine Creek Golf Course

Over the past year, 20% of the players at Pine Creek were women. In an effort to increase the proportion of women players, Pine Creek implemented a special promotion design to attract women golfers. The manager now wants to determine if the proportion of women players has increased.

A random sample of 400 players were selected and 100 of the players were women. The level of significance is $\alpha = .05$.

Tests about a Population Proportion (4 of 6)

p-value and Critical Value Approaches

1. Determine the hypotheses $H_0: p \leq .20$ and $H_a: p > .20$
2. Specify the level of significance. $\alpha = .05$
3. Compute the value of the test statistic.

$$\sigma_{\bar{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{.2(1-.2)}{400}} = .02$$

$$z = \frac{\bar{p} - p_0}{\sigma_{\bar{p}}} = \frac{(.25) - .20}{.02} = 2.50$$

Tests about a Population Proportion (5 of 6)

p-value Approach

4. Compute the *p*-value.

For $z = 2.5$, lower tail area = .9938.

$$p\text{-value} = (1 - .9938) = .0062$$

5. Determine whether to reject H_0 .

Because $p\text{-value} = .0062 < \alpha = .05$, we reject H_0 .

Using Excel to determine p -values

	A	B	C	D	E
1	Golfer		Hypothesis Test about a Population Proportion		
2	Female				
3	Male		Sample Size	=COUNTA(A2:A401)	
4	Female		Response of Interest	Female	
5	Male		Count for Response	=COUNTIF(A2:A401,D4)	
6	Male		Sample Proportion	=D5/D3	
7	Female				
8	Male		Hypothesized Value	0.2	
9	Male				
10	Female		Standard Error	=SQRT(D8*(1-D8)/D3)	
11	Male		Test Statistic z	=(D6-D8)/D10	
12	Male				
13	Male		p -value (Lower Tail)	=NORM.S.DIST(D11,TRUE)	
14	Male		p -value (Upper Tail)	=1-D13	
15	Male		p -value (TwoTail)	=2*MIN(D13,D14)	
16	Female				
400	Male				
401	Male				
402					

	A	B	C	D	E	F
1	Golfer		Hypothesis Test about a Population Proportion			
2	Female					
3	Male		Sample Size	400		
4	Female		Response of Interest	Female		
5	Male		Count for Response	100		
6	Male		Sample Proportion	0.25		
7	Female					
8	Male		Hypothesized Value	0.20		
9	Male					
10	Female		Standard Error	0.02		
11	Male		Test Statistic z	2.5000		
12	Male					
13	Male		p -value (Lower Tail)	0.9938		
14	Male		p -value (Upper Tail)	0.0062		
15	Male		p -value (TwoTail)	0.0124		
16	Female					
400	Male					
401	Male					
402						

Tests About a Population Proportion (6 of 6)

Critical Value Approach

- Determine the critical values and rejection rule.

$$z_{.05} = 1.645$$

$$\text{Reject } H_0 \text{ } z \geq 1.645$$

- Determine whether to reject H_0 .

Because $z = 2.5 > 1.645$, we reject H_0 .