4

RISK VS. RETURN

In finance, investors always consider, explicitly or implicitly, the trade-off between risk and return. Thus, we have to understand the definitions of those two terms. However, the task is not straightforward as many might think since there exist so many different definitions for both of them. For risk, we have total risk, market risk, firm specific risk, variance and standard deviation of returns, volatility, LPSD (Lower Partial Standard Deviation), and VaR (Value at Risk). In terms of frequency, standard deviation of returns could be based on daily, monthly, or annual data. For individual stock returns, we have percentage return, log returns, total return, capital gain yield, dividend yield, daily, monthly and annual returns. For mean returns, we have time-weighted, dollar-weighted, arithmetic and geometric means. For portfolio or market indices, we have value-weighted, equal-weighed or price-weighted. For the tradeoff between risk and return, we have Sharpe ratio, Treynor ratio, Sortino ratio and the trade-off based on a utility function. In particular, the following topics will be covered:

- Risk definitions: common sense and mathematical definitions
- Total risk, market and firm specific risk
- Return definitions: percentage, log return, period return, total return
- Time-weighted vs. dollar weighted
- Mean return: arithmetic mean vs. geometric means, time-weighed vs. dollar weighed
- Converting daily returns to weekly, monthly or annual ones
- Time-weighed vs. dollar-weighed
- Risk-return trade-off (I): Sharpe ratio
- Risk-return trade-off (II): Treynor ratio
- Risk-return trade-off (III): LPSD and Sortino ratio
- Risk-return trade-off (IV): Utility function

Note that VaR will not be discussed in this chapter, since we would devote a whole chapter to it: Chapter 12: VaR (Value at Risk). The different weighting schemes, such as value-weighted, equal-weighted and price weighted, will be discussed in Chapter 13: Portfolio Theory.

4.1 DEFINITIONS OF RETURN

It seems that most people understand the definitions of return or at least one of its variations. Thus, we start from return. The simplest definition of return is called percentage return. For example, we bought a stock at the price of p_0 and sell it at p_1, the return of our investment will be:

$$R = \frac{p_1 - p_0}{p_0}, \tag{1}$$

where R is the percentage return, p_1 is the selling price and p_0 is the purchasing price. For example, if we bought a share at $100 of Company A and sold it at $110, the return is 10%, (110-100)/100. In addition, if we enjoy a dividend of d_1, it should be included in our calculation:

$$R = \frac{p_1 - p_0 + d +_1}{p_0}, \tag{2}$$

where d_1 is the dividend. Actually the right hand side of the above equation could be decomposed into two parts:

$$R = \underbrace{\frac{p_1 - p_0}{p_0}}_{\substack{capital\ gain \\ yield}} + \underbrace{\frac{d_1}{p_0}}_{dividend\ yield} \tag{3}$$

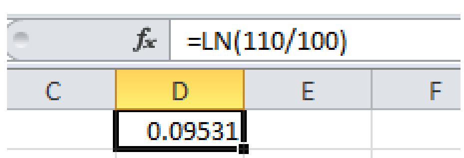
On the right-hand side, the first ratio is called capital gain return (yield) while the second term is called dividend yield, see below.

$$total\ return = capital\ gain\ yield + dividend\ yield \tag{4}$$

In the above case, if we have $1 dividend at the end of the period, the total return is 11%, 10% from the capital gain and 1% from the dividend yield. The log return is defined below.

$$R^{log} = \ln\left(\frac{p_1}{p_0}\right), \tag{5}$$

where R^{log} is the log return, the `ln()` function is the natural logarithm function. For example, the log return when p1=110 and p0=100 is 9.531%, see below.



The relations between a percentage return and log return are given below.

$$\begin{cases} R^{log} = ln(R+1) \\ R = exp(R^{log}) - 1 \end{cases} \tag{6}$$

where `exp()` is the exponential function. For example, `ln(0.1+1)` would give us 0.09531, while `exp(0.9531)-1` would give us 10%, see two images below.

f_x	=LN(0.1+1)
D	E
0.09531	

f_x	=EXP(D1)-1
D	E
0.09531018	0.1

Later, we show a good application of log return to convert daily returns to weekly/monthly/quarterly or annual ones.

4.2 ARITHMETIC MEAN VS. GEOMETRIC MEAN

Assume that we have three numbers: a, b and c. Their arithmetic mean is their summation divided by 3 while their geometric means will be product of those numbers raised to a power of 1/3. The related Excel functions are `average()` and `geomean()`. For given three values of 2, 3 and 5, the arithmetic mean is 3.333, while the geometric mean is 3.107, see the image below.

C2 | f_x =GEOMEAN(B1:B3)

	A	B	C	D	E
1	a	2	3.333333	=AVERAGE(B1:B3)	
2	b	3	3.107233	=GEOMEAN(B1:B3)	
3	c	5			

To generalize the above definitions and for n values, we have the following general formulae.

$$\bar{X}_{arithmetic} = \frac{\sum_{i=1}^{n} x_i}{n} \quad (7)$$

$\bar{X}_{arithmetic}$ is the arithmetic mean, n is the number of x variable, $\sum_{i=1}^{n} x_i$ is the summation of n numbers of xi. For example, when n is 3, $\sum_{i=1}^{3} x_i = x_1 + x_2 + x_3$. For a geometric mean, we have the following formula.

$$\bar{X}_{geomean} = (\prod_{i=1}^{n} x_i)^{\frac{1}{n}} \quad (8)$$

$\prod_{i=1}^{n} x_i$ is the product of n values of x variable. For example when n=3, $\prod_{i=1}^{n} x_i = x_1 * x_2 * x_3$. Note that the geometric mean is always less than its arithmetic mean unless x values are all equal. For that special case (all x values are all equal), those two means are equal.

For n returns, the arithmetic mean has the exact same formula, see below.

$$\bar{R}_{arithmetic} = \frac{\sum_{i=1}^{n} R_i}{n} \quad (9)$$

However, their geometric mean is quite different.

$$\bar{R}_{geomean} = [\prod_{i=1}^{n} (R_i + 1)]^{\frac{1}{n}} - 1 \quad (10)$$

For 3 given returns of 10%, 15% and -5%, their geometric mean calculation is shown below.

D1 =GEOMEAN(C1:C3)-1

	A	B	C	D	E	F
1		0.1	1.1	0.063175		
2		0.15	1.15			
3		-0.05	0.95			

From the above calculation, it is quite cumbersome to add one to teach return, then apply the formula. Why not just include all returns in our estimation? Below, a good method to calculate geometric mean for returns. The formula used in cell B7, is `=geoeanyan(c2:f2)` without adding 1 then applying the Excel `geomean()` function.

B7 =geomeanyan(C2:F2)

	A	B	C	D	E	F
1	More return, we have different ways to estimate a geometric mean					
2		return	0.010	0.030	-0.020	0.100
3		return +1	1.010	1.03	0.98	1.1
4						
5	arithmetric mean	0.03000	=AVERAGE(C2:F2)			
6	geometric mean	0.02907	=GEOMEAN(C11:F11)-1			
7	geomeanyan()	0.02907	=geomeanyan(C2:F2)			

Interested readers could download the Excel file from the author's website at http://canisius.edu/~yany/excel/geomeanYan.xlsm. Alternatively, when studying Chapter 28: Simple VBAs, type `.c28(10)` to find the related VBA.

4.3 TIME-WEIGHTED VS. DOLLAR-WEIGHTED

When estimate a multiple-period return, we use buy-and-hold methodology. Here is a simple 2-period example from Dichev (2007). The following time line shows the prices of stock A at three points-in-time, see the image below.

C	D
Time	stock price
0	10
1	20
2	10

At T_0, the price of a stock is $10 and an investor purchases 100 shares. At T_1, the stock price jumps to $20, she buys another 100 shares. At the end of her investment horizon, T_2, the price falls back to $10 and at this moment she sells all of her holding, see the cash flow with the image below.

fx =-D2*E2

C	D	E	F
Time	stock price	holding	cash flow
0	10	100	-1000
1	20	100	-2000
2	10	-200	2000

What is the total return? According to finance textbooks, the total return is zero, see below.

$$R = [(1+100\%) * (1-50\%)]^{\frac{1}{2}} - 1 = 0 \text{ or } (10\text{-}10)/10\text{=}0$$

Unfortunately, this is correct for an investor who has cash flows at T_0 and T_2 only. For our investor, she has cash flows at T_0, T_1 and T_2. Her cash outflow is $3,000 ($1,000 at T_0 and $2,000 at T_1) and her cash inflow is only $2,000 (at T_2). Without calculation, we know that her total return should be negative: $3,000 cash outflow and $2,000 cash inflow even without considering the time value of money. Dichev (2007) argues that the timing of the cash flows matters. The correct answer should be -26.8%. Dichev suggests that we should use dollar weighted total return which is the same as an IRR (Internal Rate of Return), `=IRR(-1000, -2000, 2000)=-26.8%`.

G2 fx =IRR(F2:F4)

	A	B	C	D	E	F	G
1			Time	stock price	holding	cash flow	
2			0	10	100	-1000	-26.795%
3			1	20	100	-2000	
4			2	10	-200	2000	

4.4 DEFINITIONS OF RISK

If limited to one word to describe risk, it is "uncertainty". There are many ways to define risk, see the summary in the following table. The market risk will be discussed in Chapter A: CAPM (Capital Asset Pricing Model), while VaR will be discussed in Chapter 12: VaR (Value at Risk).

Table 4.1 Definitions of Risk

#	Greek letter	Description	Chapter
1	σ^2	Variance of returns	4
2	σ	Standard deviation of returns	4
3	LPSD	Lower Partial Standard Deviation	4
4	β	Market risk	7
5	VaR	Value at Risk	12

Method I: when a set of probabilities plus their related returns are given, we estimate the mean first, see the formula below.

$$E(R) = \bar{R} = \sum_{i=1}^{n} p_i R_i \tag{11}$$

where E(R) is the expected value and we use R to represent it. n is the number of securities, $\sum_{i=1}^{n} x_i$ is the summation, R_i is the i^{th} return of the underlying security. For the related variance, we have the following formula:

$$\sigma^2 = \sum_{i=1}^{n} p_i (R_i - \bar{R})^2 \tag{12}$$

where σ^2 is the variance.

Method II: for a given individual stock or portfolio, the definition of a variance of returns is given below.

$$\sigma^2 = \frac{\sum_{i=1}^{n}(R_i - \bar{R})^2}{n-1} \tag{13}$$

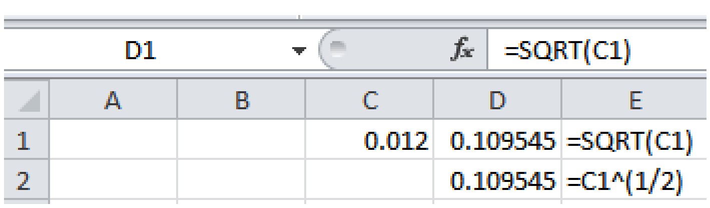
where σ^2 is the variance, R_i is the i^{th} return of the underlying security, $\sum_{i=1}^{n} x_i$ is the arithmetic mean and n is the number of returns. The above formula is related to sample. For the population, we have the following formula.

$$\sigma^2 = \frac{\sum_{i=1}^{n}(R_i - \bar{R})^2}{n} \tag{14}$$

The only difference, between the above two equations, is the denominators: for a sample, it is n-1, while it is n for the population. Obviously, the n is large enough; there is no difference between those two types of definitions. The standard deviation is the square root of a variance.

$$\sigma = \sqrt{\sigma^2} \tag{15}$$

For Excel, we could use the `sqrt()` function of the power of ½, see the image below.



Both variance and standard deviation are used to describe volatility. However, volatility is usually referred to the standard deviation. For example, the volatility of IBM is 20% means that the annualized standard deviation of its return is 20%.

4.5 TOTAL RISK VS. MARKET RISK

The variance and standard deviation of return are measures of total risk. In the above table, we know that beta is a measure of market risk. The relationship between those two are given below.

$$total\ risk = market\ risk\ + firm\ specific\ risk \tag{16}$$

At this moment, we just mention the above relationship first. In Chapter 12: Portfolio Theory, we would elaborate this concept in more detail.

Table 4.2: Other names for market and firm specific risks

Market risk	Firm specific risk
Common risk	Unique risk
Systematic risk	Unsystematic risk
Undiversifiable risk	Diversifiable risk

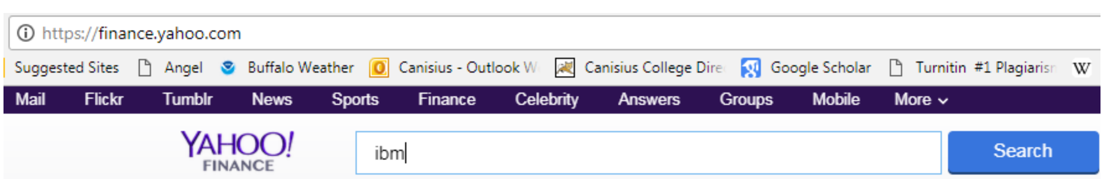
4.6 EXCEL FUNCTIONS TO CALCULATE σ^2 AND σ

The following table lists several Excel functions related to estimation of variance and standard deviation.

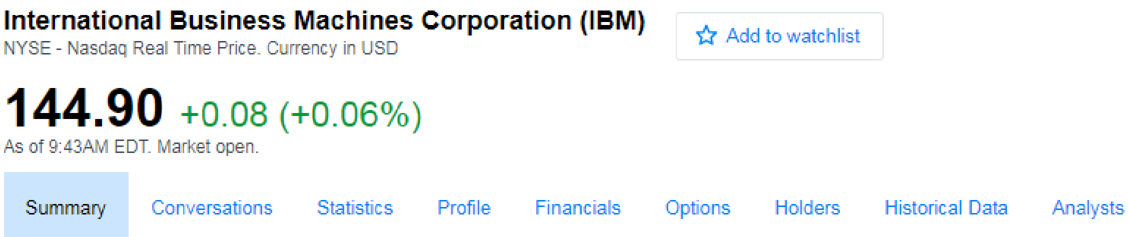
Table 4.3: Excel functions for the estimations of σ2 and σ

#	Function	Population or sample
1	=var.p()	Population
2	=var.s()	Sample
3	=var()	Sample
4	=stdev.p()	Population
5	=stdev.s()	Sample
6	=stdev()	Sample

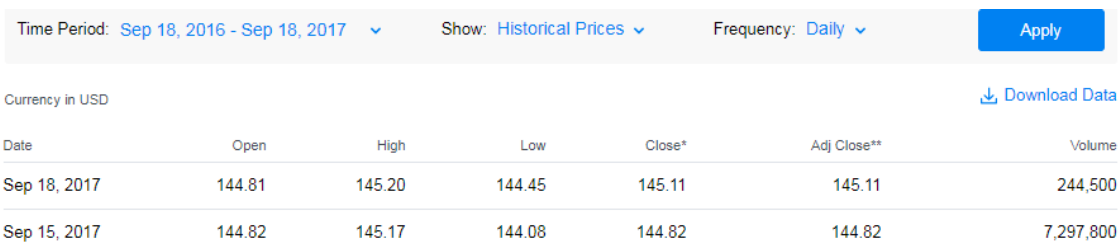
Below is a simple example. First, from http://finance.yahoo.com,



After enter “ibm”, we hit Search on the right-hand side, see the following image.



After clicking “Historical Data”, see below.



Date	Open	High	Low	Close*	Adj Close**	Volume
Sep 18, 2017	144.81	145.20	144.45	145.11	145.11	244,500
Sep 15, 2017	144.82	145.17	144.08	144.82	144.82	7,297,800

By choosing appropriate time period and monthly frequency, we download the IBM’s data. Using Excel, we open the saved file, the first few lines are shown below.

	A	B	C	D	E	F	G
1	Date	Open	High	Low	Close	Adj Close	Volume
2	9/1/2012	null	null	null	null	null	null
3	10/1/2012	208.01	211.79	190.56	194.53	168.2367	96155000
4	11/1/2012	194.68	198	184.78	190.07	164.3796	81892500
5	12/1/2012	190.76	196.45	186.94	191.55	166.3845	79320200
6	1/1/2013	194.09	208.58	190.39	203.07	176.3911	87636600
7	2/1/2013	204.65	205.35	197.51	200.83	174.4453	64144100
8	3/1/2013	200.65	215.9	199.36	213.3	186.0569	76023800

The *Date* column, Column A, is for date, *Open* is the opening price of the period, *High* is the highest price achieved during the period, *Low* is the lowest price achieved during the period, *Close* is the closing price, *Volume* is the trading volume and *Adj.Close* is the adjusted closing price. When estimating returns, we have to use adjusted closing price instead of close price since the former considers stock split, dividend and other distributions.

Adj.Close is used to estimate the monthly returns, see below. Please pay attention to the sorting of the observations: from the latest to the oldest. When typing the formula, =F4/F3-1 is better than `=(F4-F3)/F3` since the former would avoid errors caused by a typo.

H4 f_x =(F4-F3)/F3

	A	B	C	D	E	F	G	H
1	Date	Open	High	Low	Close	Adj Close	Volume	R (IBM)
2	9/1/2012	null	null	null	null	null	null	
3	10/1/2012	208.01	211.79	190.56	194.53	168.2367	96155000	
4	11/1/2012	194.68	198	184.78	190.07	164.3796	81892500	-0.02293
5	12/1/2012	190.76	196.45	186.94	191.55	166.3845	79320200	

Hit Ctrl-End, we could find the last record, see the image below. Since there is no record after row 62, we would see an effort message for the last return.

59	6/1/2017	152.8	157.2	150.8	153.83	152.2217	83977000	0.017708
60	7/1/2017	153.58	156.03	143.64	144.67	143.1575	93271700	-0.05955
61	8/1/2017	145	145.67	139.13	143.03	141.5346	80268700	-0.01134
62	9/1/2017	142.98	146.38	141.64	144.82	144.82	42262600	0.023213

By applying Excel functions related to variance, standard deviation, we could calculate their values, see the image below.

I3 f_x =VAR.P($H$4:$H$62)

	A	B	C	D	E	F	G	H	I	J	K
1	Date	Open	High	Low	Close	Adj Close	Volume	R (IBM)			
2	9/1/2012	null	null	null	null	null	null				
3	10/1/2012	208.01	211.79	190.56	194.53	168.2367	96155000		0.00253423	=VAR.P($H$4:$H$62)	
4	11/1/2012	194.68	198	184.78	190.07	164.3796	81892500	-0.02293	0.00257792	=VAR.S($H$4:$H$62)	
5	12/1/2012	190.76	196.45	186.94	191.55	166.3845	79320200	0.012197	0.00257792	=VAR($H$4:$H$62)	
6	1/1/2013	194.09	208.58	190.39	203.07	176.3911	87636600	0.060141	0.05034115	=STDEV.P($H$4:$H$62)	
7	2/1/2013	204.65	205.35	197.51	200.83	174.4453	64144100	-0.01103	0.05077327	=STDEV.S($H$4:$H$62)	
8	3/1/2013	200.65	215.9	199.36	213.3	186.0569	76023800	0.066563	0.05077327	=STDEV($H$4:$H$62)	

Again, we should pay attention to the last record since it has an error. From the image above, we could see that the standard deviation based on the IBM's last 5-year monthly returns is 0.05077327. In the next section, we show how to annualized this standard deviation. The Excel `var()` function is the same as `var.s()` and `stddev()` is the same as `stdev.s()`. Optionally, we could make our lives a little easier by using the absolute address. First, for cell J2, we estimate variance based on the population. After we get our value, fix the range by using absolute addresses. Copy the formula to the next five cells. Then change their formula without touching the range.

4.7 ANNUALIZING σ² AND σ

When asking about salary, the convention is for annual ones. This is true for the volatility and it should be annualized ones. However, most of time, we have historical monthly or daily returns. The following formula are used to convert to the annual ones. From monthly variance and standard deviation to annual ones, we have the following formulae.

$$\begin{cases} \sigma^2_{annual} = 12\sigma^2_{monthly} \\ \sigma_{annual} = \sqrt{12}\,\sigma_{monthly} \end{cases} \tag{17}$$

For the example in the previous section, the standard deviation (volatility) based on the monthly returns is 5.0773%. Thus, its corresponding annualized standard deviation or volatility is 17.6%, see the image below.

J9 | fx =I8*SQRT(12)

	A	B	C	D	E	F	G	H	I	J	K
1	Date	Open	High	Low	Close	Adj Close	Volume	R (IBM)			
2	9/1/2012	null	null	null	null	null	null				
3	10/1/2012	208.01	211.79	190.56	194.53	168.2367	96155000		0.00253423	=VAR.P($H$4:$H$62)	
4	11/1/2012	194.68	198	184.78	190.07	164.3796	81892500	-0.02293	0.00257792	=VAR.S($H$4:$H$62)	
5	12/1/2012	190.76	196.45	186.94	191.55	166.3845	79320200	0.012197	0.00257792	=VAR($H$4:$H$62)	
6	1/1/2013	194.09	208.58	190.39	203.07	176.3911	87636600	0.060141	0.05034115	=STDEV.P($H$4:$H$62)	
7	2/1/2013	204.65	205.35	197.51	200.83	174.4453	64144100	-0.01103	0.05077327	=STDEV.S($H$4:$H$62)	
8	3/1/2013	200.65	215.9	199.36	213.3	186.0569	76023800	0.066563	0.05077327	=STDEV($H$4:$H$62)	
9	4/1/2013	212.8	214.89	187.68	202.54	176.6712	108666600	-0.05045	annualized	0.175884	

Similarly, from daily variance and standard deviations, we have the following formulae.

$$\begin{cases} \sigma^2_{annual} = 252\sigma^2_{daily} \\ \sigma_{annual} = \sqrt{252}\,\sigma_{daily} \end{cases} \tag{18}$$

4.8 DAILY RETURNS TO WEEKLY/MONTHLY/ANNUAL ONES

Below, we show how to use log returns and Pivot Table to estimate the mean and standard deviation of n-day returns. The key logic is that the log return of a longer period is the summation of log return of its sub-periods.

$$R^{log}_{longPeriod} = \sum_{i=1}^{n} R^{log}_i \tag{19}$$

Where $R^{log}_{longPeriod}$ is the longer-term log return, R^{log}_i is the i^{th} sub period's log return and n is the number of the sub-periods within the long period. Here is an example. Assume that there are 20 trading days within one month. A monthly log return is the summation of those 20 daily log returns. Similarly, an annual log return is the summation of 4 quarterly log returns, as well the summation of 12 monthly log returns or the summation of 52 weekly log returns or 252 daily log returns. This property makes our conversion, between different frequencies, extremely easy.

Below, assume we are interested in an annual return by using daily price data. Here is the procedure.

Step 1: Download daily price data
Step 2: Estimate daily log return by applying the `ln()` function
Step 3: Generate a YYYY column
Step 4: Click "Insert" on the Excel menu bar, then choose "Pivot Table". Select the *Period* variable as "Row Labels" and the daily log daily returns as "Values". Since the summation of daily log returns is annual log return, choose "Sum of daily log returns" in the setting of the "Values".
Step 4B: (optional) Copy and paste the annual log returns to a new spreadsheet.
Step 6: Apply the following formula to convert an annual log return to a percentage return.

$$R = e^{R^{log}} - 1 \qquad (20)$$

Similarly, we could convert daily returns to weekly, monthly, or quarterly ones. Below are a few simple steps:
Step 1: download daily price data from Yahoo!Finance. A reader could manually do so. Alternatively, he/she could issue the following two lines.

```
> x=.getDailyPrice("ibm")
> .saveYan(x,"c:/temp/ibmDaily.csv")
[1] "Your saved file is ==>c:/temp/ibmDaily.csv"
```

The first line downloads monthly price data for IBM. The only input variable is the ticker symbol, "ibm" in this case. Since this is a string variable, we have to include it in a pair of double or single quotation marks. Note that we assign all observations to x which a place holder. If omitting "x=", all observations should show on our screens. The second line saves our output to our output file. The third line reminds users the location of the saved file.

Step 2: Estimate daily log return, see the formula in H2.

H2 fx =LN(G2/G3)

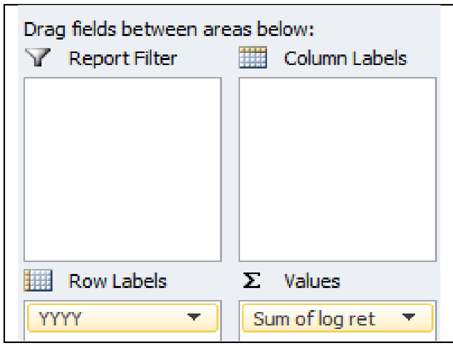
	A	B	C	D	E	F	G	H
1	Date	Open	High	Low	Close	Volume	Adj.Close	log ret
2	4/7/2017	172.08	172.93	171.28	172.14	3556000	172.14	-0.0018
3	4/6/2017	173.47	173.47	172.25	172.45	3416100	172.45	-0.00249
4	4/5/2017	174.7	176.33	172.81	172.88	6199500	172.88	-0.00944

Step 3: Generate a column of YYYY by applying the Excel `year()` function, see the formula in I2.

I2 fx =YEAR(A2)

	A	B	C	D	E	F	G	H	I
1	Date	Open	High	Low	Close	Volume	Adj.Close	log ret	YYYY
2	4/7/2017	172.08	172.93	171.28	172.14	3556000	172.14	-0.0018	2017
3	4/6/2017	173.47	173.47	172.25	172.45	3416100	172.45	-0.00249	2017
4	4/5/2017	174.7	176.33	172.81	172.88	6199500	172.88	-0.00944	2017

Step 4: Click “Insert” on the menu bar, then “PivotTable”. Choose appropriate range and select YYYY as “Row Labels” and log ret as “Σ Values”, see the left image below. The final result is shown in the right image below.



	A	B
1		
2		
3	Row Labels	Sum of log ret
4	1962	-0.382290653
5	1963	0.263303405
6	1964	0.010624078
7	1965	0.198912474
8	1966	0.111646032

Step 5: we could sort the “Row Labels” from the latest to the oldest. Copy those values to another spread sheet and covert those annual log returns into annual percentage returns by applying the following converting formula.

$$R = e^{R^{log}} - 1 \quad (21)$$

where R is our percentage return, R^{log} is the log return. The related converting formula is shown in cell C2.

C2 f_x =EXP(B2)-1

	A	B	C	D
1	Row Label	Sum of log ret	ret (%)	
2	1962	-0.382290653	-0.3177	
3	1963	0.263303405	0.301221	
4	1964	0.010624078	0.010681	
5	1965	0.198912474	0.220075	

4.9 TRADEOFF (I): SHARPE RATIO

The Sharpe ratio is defined below.

$$\text{Sharpe} = \frac{\bar{R}_p - R_f}{\sigma_p} \quad (22)$$

Where $\bar{R}_p$ is the mean portfolio returns, R_f is the risk free rate, σ_p is the risk, standard deviation, of the portfolio. The Sharpe ratio could be interpreted as the benefit cost ratio: the benefit is the portfolio risk premium while the cost is the total risk.

J8 | f_x | =(J5-J3)/J6

	A	B	F	G	H	I	J
1	Date	Open	Volume	Adj.Close	ret		
2	4/3/2017	173.82001	6589300	172.88	-0.00724	risk-free	0.02
3	3/1/2017	180.48	3538400	174.14	-0.03159	risk-free (monthly)	0.001667
4	2/1/2017	175	3291100	179.82	0.038516		
5	1/3/2017	167	4644900	173.1509	0.051389	mean	0.000451
6	12/1/2016	161.95	3448700	164.6878	0.02324	std	0.050125
7	11/1/2016	153.5	3844600	160.9474	0.065077		
8	10/3/2016	158.06	3899000	151.1134	-0.03248	sharpe ratio	-0.02425

4.10 TRADEOFF (II): TREYNOR RATIO

Instead of using the total risk as our cost, Treynor ratio uses the market risk instead, see its related formula below.

$$\text{Treynor} = \frac{\bar{R}_p - R_f}{\beta_p} \tag{23}$$

where β_p is the market risk (beta) of our portfolio. Again, the only difference, between the above two ratios, is related to their denominators: using total risk vs. using market risk. This difference determines which ratio should apply under different situations. When the underlying portfolio is our all wealth, then Sharpe ratio is appropriate. On the other hand, if the portfolio under consideration is only part of our total wealth, the firm specific risk should not be considered. Thus, the Treynor ratio is a preferred measure. Assume that IBM's beta is 0.8. Based on the above data set, we could estimate the Treynor ratio, see the image below.

J9 | f_x | =(J5-J3)/J7

	A	B	F	G	H	I	J
1	Date	Open	Volume	Adj.Close	ret		
2	4/3/2017	173.82001	6589300	172.88	-0.00724	risk-free	0.02
3	3/1/2017	180.48	3538400	174.14	-0.03159	risk-free (monthly)	0.001667
4	2/1/2017	175	3291100	179.82	0.038516		
5	1/3/2017	167	4644900	173.1509	0.051389	mean	0.000451
6	12/1/2016	161.95	3448700	164.6878	0.02324	std	0.050125
7	11/1/2016	153.5	3844600	160.9474	0.065077	beta=	0.8
8	10/3/2016	158.06	3899000	151.1134	-0.03248	sharpe ratio	-0.02425
9	9/1/2016	158.32001	3501600	156.1869	-0.00019	Treynor	-0.00152

4.11 TRADEOFF (II): LPSD AND SORTINO RATIO

Sortino argues that when estimating a standard deviation based on stock returns, both positive and negative deviation from the mean are considered and treated equally. However, this is not correct since in the real-world, investors are less concerned with the positive deviation. Because of this, we could consider those returns that are less than a benchmark. Here is the formula to estimate LPSD if we use risk-free rate as our benchmark.

$$LPSD = \sqrt{\frac{\sum_{i=1}^{n}(R_i - R_f)^2}{n-1}} \quad if\ R_i < R_f \qquad (24)$$

Where LPSD is Lower Partial Standard Deviation, R_i is the i^{th} stock return, Rf is tour benchmark and n is the number of returns. Note that if we have only m returns having a value less than rf, the summation is based on those m values. The second inconsistence for Sharpe ratio is that for the numerator the benchmark is the risk-free rate, i.e., risk premium, while the benchmark is the mean return. If we use risk-free rate for both, we have the following Sortino ratio.

$$Sortino = \frac{\bar{R} - \bar{R}_f}{LPSD} \qquad (25)$$

Using the same data, download,

```
> x=.getDailyPrice("ibm")
> .saveYan(x,"c:/temp/ibmDaily.csv")
[1] "Your saved file is ==>c:/temp/ibmDaily.csv"
```

We could estimate LPSD and Sortinor ratio. For the formula in K2, we pick return when its value is less than our bencher mark i.e., risk-free in this case. Then we estimate their squared values, see column L. The square root of the summation of those squared value divided by n-1 will be our LPSD, see the formula in N3. By applying the above formula, we could get our Sortino ratio of -0.01, see the value and formula in N4.

K2   fx =IF(H2>$J$3,0, H2)

	A	H	I	J	K	L	M	N	O	P	Q
1	Date	ret			R>Rf?	(R-Rf)^2	mean	0.0004511	=AVERAGE(H2:H61)		
2	4/3/2017	-0.00724	Rf (annual)	0.01	-0.00724	5.23528E-05	sum	0.0714831	=SUM(L2:L61)		
3	3/1/2017	-0.03159	Rf (monthly)	0.00083	-0.03159	0.00099775	LSPD	0.0348077	=SQRT(N2/(COUNT(H2:H61)-1))		
4	2/1/2017	0.038516			0	0	Sortino	-0.0109817	=(N1-J3)/N3		
5	1/3/2017	0.051389	stdev=	0.05013	0	0	Sortino(VBA)	-0.0108373	=sortinoRatio(H2:H61,J3)		
6	12/1/2016	0.02324			0	0					

Alternatively, we could apply the VBA to get the ratio. The related VBA is listed in Appendix A. Again, at this moment, no requirement is for understanding VBA. For interested readers, please read chapter 28: Simple VBAs.

4.12 TRADEOFF (IV): A UTILITY FUNCTION

In our real world, many things we could quantify such as one table, two bikes and a five-dollar bill. However, many things we could not quantify such as happiness, sadness and regrets. Trying to quantify them, economists have developed a concept called utility. The values of a utility function are not import. The properties of a utility function are important. For example, when we earn more money, our utility increases. This is understandable. On the other hand, the benefit for next extra dollar will be smaller than the benefit from the previous dollar. Just image a young man just started his career and got a bonus check of $500. He would get very excited. However, when his salary or wealth has increased over the years, the same $500 would bring him less joy. Below is a utility function for the tradeoff between risk and return.

$$U = E(R) - \frac{1}{2}A\sigma^2 \tag{26}$$

Where U is the utility, E() is the expectation, R is the return of the underlying security or the portfolio, A is a risk-averse factor and σ^2 is the variance of the underlying security or portfolio. Since return is treated as benefit, U is an increasing function of benefit, R. Similarly, because of negative sign, U is the decreasing function of the cost, expressed by the variance of our returns. Compared with other risk-return tradeoff measures, the risk-averse of A is a new variable here. Below, we use the SMB from Fama-French's monthly factor as our portfolio to illustrate the concept of indifference curve. First, we issue the following two lines to download the data.

```
> .showff3Monthly(0)
Launch Excel and paste
```

Note that the last line above tells us the location of the output file. The first several lines of the monthly Fama-French monthly factors are shown below.

G9 | =G5-0.5*G8*G6^2

	A	B	C	D	E	F	G	H	I
1	DATE	MKT_RF	SMB	HML	RF				
2	7/1/1926	0.0296	-0.023	-0.0287	0.0022	mean	0.002115	=AVERAGE(C2:C1080)	
3	8/1/1926	0.0264	-0.014	0.0419	0.0025	std	0.03218	=STDEV(C2:C1080)	
4	9/1/1926	0.0036	-0.0132	1.00E-04	0.0023				
5	10/1/1926	-0.0324	4.00E-04	0.0051	0.0032	mean (annual)	0.025673	=(1+G2)^12-1	
6	11/1/1926	0.0253	-0.002	-0.0035	0.0031	std (annual)	0.111475	=SQRT(12)*G3	
7	12/1/1926	0.0262	-4.00E-04	-2.00E-04	0.0028				
8	1/1/1927	-6.00E-04	-0.0056	0.0483	0.0025	A	1		
9	2/1/1927	0.0418	-0.001	0.0317	0.0026	Utility	0.01946	=G5-0.5*G8*G6^2	

The annualized return and standard deviation of SMB (Small minus Big) are 2.5673% and 11.1475%, respectively. If we assume that A has a value of 1, the utility function for an investor who choose Fama-French SMB as his/her portfolio will be 0.01946. Note that the absolute value of utility is not important. The relative value matters.

4.13 CONCEPT OF AN INDIFFERENCE CURVE

When discussing risk-return tradeoff, we have to discuss the indifference curve. Let's use an analogy. John is earning $50,000 per year. The head-hunter sent him an invitation for another job with a higher salary of $55,000. However, since that job is located in another city, it is quite convenient for John. After careful consideration of many factors, John made a counter offer of $58,000. This value will be John's indifference salary for the new job compared with his current job.

In short, an indifference curse, drawn with risk-as x-axis and return as y-axis, gives the equivalent combinations of risk-and return for a specific investor. For Sharpe, Treynor and Sortino ratios, their indifference curses are a straight line. Using Sharpe ratio, see below, as an example. When both risk-premium and risk doubled, the Sharpe remains the same. This is true for Treynor and Sortino ratios.

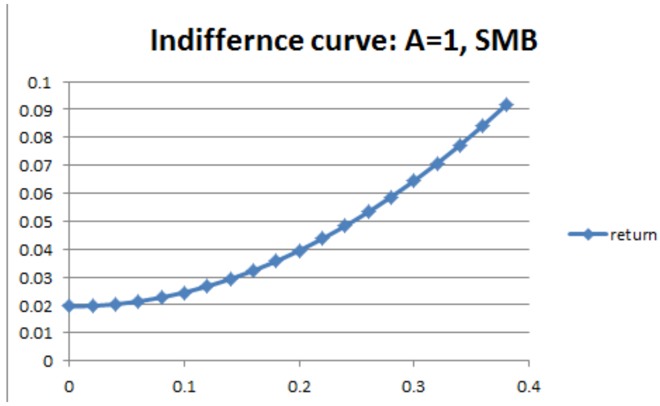
$$\text{Sharpe} = \frac{\bar{R}_p - R_f}{\sigma_p}$$

We use the utility value of 0.01946 for the SMB monthly factor shown in the previous section as an example. First, we generate a column called risk which talks value of 0, 0.02, 0.04, and so on with an incremental value of 2%. Then we estimate the expected value that makes a constant of utility of 0.01946, see column K, especially the formula in K5.

K5 fx =$G$9+0.5*$G$8*J5^2

	F	G	J	K	L
1				utility	
2	mean	0.002115			
3	std	0.03218			
4			risk	return	
5	mean (annual)	0.025673	0	0.01946	
6	std (annual)	0.111475	0.02	0.01966	
7			0.04	0.02026	
8	A	1	0.06	0.02126	
9	Utility	0.01946	0.08	0.02266	
10			0.1	0.02446	
11			0.12	0.02666	
12			0.14	0.02926	
13			0.16	0.03226	
14			0.18	0.03566	
15			0.2	0.03946	
16			0.22	0.04366	
17			0.24	0.04826	
18			0.26	0.05326	
19			0.28	0.05866	
20			0.3	0.06446	
21			0.32	0.07066	
22			0.34	0.07726	
23			0.36	0.08426	
24			0.38	0.09166	

The related graph is shown below.



SUMMARY

In this chapter, before discussing a trade-off between risk and return, we disucss many different definitions of risk and returns. For example, for risk, we have total risk, market risk, firm specific risk, LPSD (Lower Partial Standard Deviation), VaR (Value at Risk, will be discussed in Chapter 12:VaR (Value at Risk). For the tradeoff between risk and return, we have Sharpe ratio, Treynor ratio and Sortino ratio.

In the next chapter, Chapter 5: T-test, F-test, tests of equal-means and test of equal variances, we would answer a few questions. Is the risk, measured by standard deviation, a constant over the time? How to answer a simple question: what is the expected annual return for IBM next year? Assume that we have last 50 years annual returns for the company. Could we simply offer the mean as our answer? Here is another example. How to claim that two stocks have different risk levels? In many cases, we have to conduct many certain types of tests to confirms or reject our hypotheses or before we offer a simple answer.

References

Dichev, Ilia D, 2007, What are stock investors' actual historical returns? Evidence from dollar-weighted returns, *American Economic Review* 97, 386-401.

Sharpe, William F.,1994, The Sharpe Ratio, The Journal of Portfolio Management 21,1,49–58. Sharpe, W. F., 1966, Mutual Fund Performance, *Journal of Business* 39, S1.

Sortino, F.A., Price, L.N., 1994, Performance measurement in a downside risk framework, *Journal of Investing* 3,50–8.

Appendix A: Explanation for chapter 4: Risk vs. return. Just type `.c4`.

```
> .c4
function(i){
" i  Chapter 4: Risk vs. return
  -  ------------------------------------
  1  What is risk (a common sense definition)
  2  mathematical definition of risk
  3  formula 4 variance/standard deviation with probability
  4  VBA 4 meanGivenProb
  5  VBA 4 varGivenProb
  6  formula 4 variance/standard deviation with historical returns
  7  total risk vs. market risk
  8  log function vs. exponential functions
  9  definition of returns
 10  continuously compounded returns (interest rate)
 11  Rc to an effective rate or APR (annual percentage rates)
 12  Excel functions for natural log and log with other bases
 13  Sharpe  ratio and Treynor ratio
 14  LPSD and Sortino ratio
 15  VBA for LPSD
 16  Time-weighed vs. dollar-weighted
 17  What is the 'Book to market Effect'
 18  risk-free rate
 19  What is the 'Small Firm Effect'
 20  Links

 Example #1:>.c4    # see the above list
 Example #2:>.c4(1) # see the first explanation
```

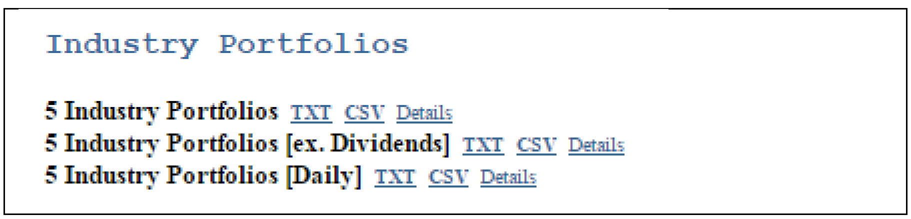
Appendix B: Data Case #5: which industry portfolio do investors prefer?
Objectives:

1) Understand the definition of and how to form 5 industries
2) Learn how to download data from Prof. French's Data Library
3) Understand the utility function, see Equation (1) below.
4) Find out which industry is optimal for different types of investors (A=1, 3 and 7) or other types of risk-preferences
5) Learn how to draw an indifference curve (for just one optimal portfolio)

Procedure:

Step 1: Go to Professor French's Data Library at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Step 2: Click CSV on the right-hand side of 5 Industry Portfolio, see the image below.


Industry Portfolios

5 Industry Portfolios TXT CSV Details
5 Industry Portfolios [ex. Dividends] TXT CSV Details
5 Industry Portfolios [Daily] TXT CSV Details

Step 3: estimate returns and variances for both value-weighted and equal-weighed industry portfolios

Step 4: Estimate the utility function for tree types of investors with A=1, 3 and 7.

$$U = E(R) - \frac{1}{2}A * \sigma^2 ,$$

where U is the utility function, E(R) is the expected portfolio return and we could use its mean to approximate, A is the risk-averse coefficient and σ^2 is the variance of the portfolio.

Step 5: Choose one result, e.g., the optimal value-weighted portfolio for investor who has a value of 1 for A, draw an indifference curve

Step 6: Comments on your results.

Note: for the definitions of those 5 industries, see the following link.
http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/det_5_ind_port.html

Appendix C: Data Case #6: Comparison between Sharpe ratio and Sortino ratio

Objectives:

1) Understand the definition of Sharpe ratio
2) Understand the definition of LPSD (Lower Partial Standard Deviation) and Sortino ratio
3) Learn how to download data from Prof. French's Data Library
4) Learn how to process data to estimate LPSD manually
5) Learn how to copy-and-paste VBA to estimate LPSD

Computational tool: Excel

Procedure:

Step 1: Go to Professor French's Data Library at
http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Step 2: Click CSV for Fama/French 3 factors, see the image below.

Fama/French 3 Factors TXT CSV Details
Fama/French 3 Factors [Weekly] TXT CSV Details
Fama/French 3 Factors [Daily] TXT CSV Details

Step 3: estimate returns and standard deviation and LPSD of three portfolio: Market, SMB (Small Minus Big) and HML (High Minus Low)
Note: using both ways (manually and VBA) to estimate LPSD

Step 4: Estimate Sharpe and Sortino ratios according to the following formulae:

$$\text{Sharpe} = \frac{\bar{R}_p - R_f}{\sigma_p} \quad (1)$$

where Sharpe is the Sharpe Ratio, $\bar{R}_p$ is the mean of our underlying portfolio, Rf is the risk-free rate, σ_p is the standard deviation of the portfolio.

$$\text{Sortino} = \frac{\bar{R}_p - R_f}{LPSD} \quad (2)$$

where LPSD is called Lower Partial Standard Deviation which is defined below.

$$LPSD = \sqrt{\frac{\sum_{i=1}^{n} (R_i - R_f)^2}{n-1}} \quad if\ R_i < R_f \quad (3)$$

Step 5: Comments on your results.

Appendix D: VBA for LPSD

Note that at this moment, it is not required to understand how to use this VBA. Interested readers could read Chapter28: Simple VBAs. Eventually, it is just copy-and-paste since this book will not teach VBA.

```
Function LPSD(returns As Range, MAR As Variant) As Variant

Dim n As Integer
Dim i As Integer
Dim avgReturn As Double
Dim mm2d As Double
Dim downDev As Double

n = returns.Rows.Count
avgReturn = WorksheetFunction.Average(returns)
moment2d = 0
For i = 1 To n
   If returns(i) - MAR < 0 Then
     mm2d = mm2d + ((returns(i) - MAR) ^ 2)
   End If
Next
downDev = Sqr(mm2d / (n - 1))
If downDev > 0 Then
   LPSD = downDev
Else
   LPSD = \"undefined\"
End If

End Function
```

Note: the above VBA could be downloaded at http://canisius.edu/~yany/excel/LPSD.txt

Appendix E: VBA for meanGivenProb and VBA for VarGivenProb.

```
Function meanGivenProb(prob As Range, returns As Range) As Variant
'by yany@canisius.edu 8/31/2017

Dim n As Integer
Dim i As Integer
Dim mean As Double
n = returns.Rows.Count
mean = 0
For i = 1 To n
    mean = mean + prob(i) * returns(i)
Next
meanGivenProb = mean

End Function
```

```
Function varGivenProb(prob As Range, returns As Range) As Variant
'by yany@canisius.edu 8/31/2017
Dim n As Integer
Dim i As Integer
Dim mean As Double
Dim final As Double

n = returns.Rows.Count
mean = 0
For i = 1 To n
    mean = mean + prob(i) * returns(i)
Next
final = 0
For i = 1 To n
    final = final + prob(i) * (returns(i) - mean) ^ 2
Next
varGivenProb = final
End Function
```

EXERCISES

4.1 How to define return? What is the difference between total return and period return?

4.2 How to calculate a total return? For example, we have 12 monthly returns. How to calculate annual return?

4.3 What is the difference between arithmetic mean and geometric mean?

4.4 What are the Excel functions for arithmetic mean and geometric mean?

4.5 For IBM, what is its total risk?

4.6 What is the difference between total risk and market risk? Which risk gets compensated and why?

4.7 What is the definition of an indifference curve?

4.8 Could we have various indifference curves for Sharpe ratio or Treynor ratio?

4.9 How many ways to measure risk for a specific firm, such as Walmart?

4.10 What are the difference between Sharpe ratio and Treynor? Which one is “better” ?

4.11 What is the definition of LPSD? Does its calculate related to the mean of the stock returns?

4.12 What does it mean “indifference curve”?

4.13 Are the indifference curses the same for different investor with same A? Note A is the risk-averse factor in their utility function.

4.14 Are the indifference curses the same for different investor with different A?

4.15 What is the indifference curve for Fama-Fench HML and market portfolios? You can assume that the risk aversion factor A is 1. Note that you can use the following two lines to download data.

```
> x=.showff3Monthly(0)
> .saveYan(x,"c:/temp/ffMonthly.csv")
```

4.16 How to show graphically that standard deviations, volatilities, of individual stocks are more volatile than the market, such as the S&P500 index? Note that ticker for the S&P500 is ^GSPC for Yahoo!Finance.

4.17 By using past 10 years’ monthly data for IBM, draw an indifference curve based on the following utility function with A=2.

$$U = E(R) - \frac{1}{2}A\sigma^2$$

4.18 What are the total return over the past 10 years by using both Fama-French daily and monthly data. Are they the same? If not, what might be the reasons cause the differences.