6

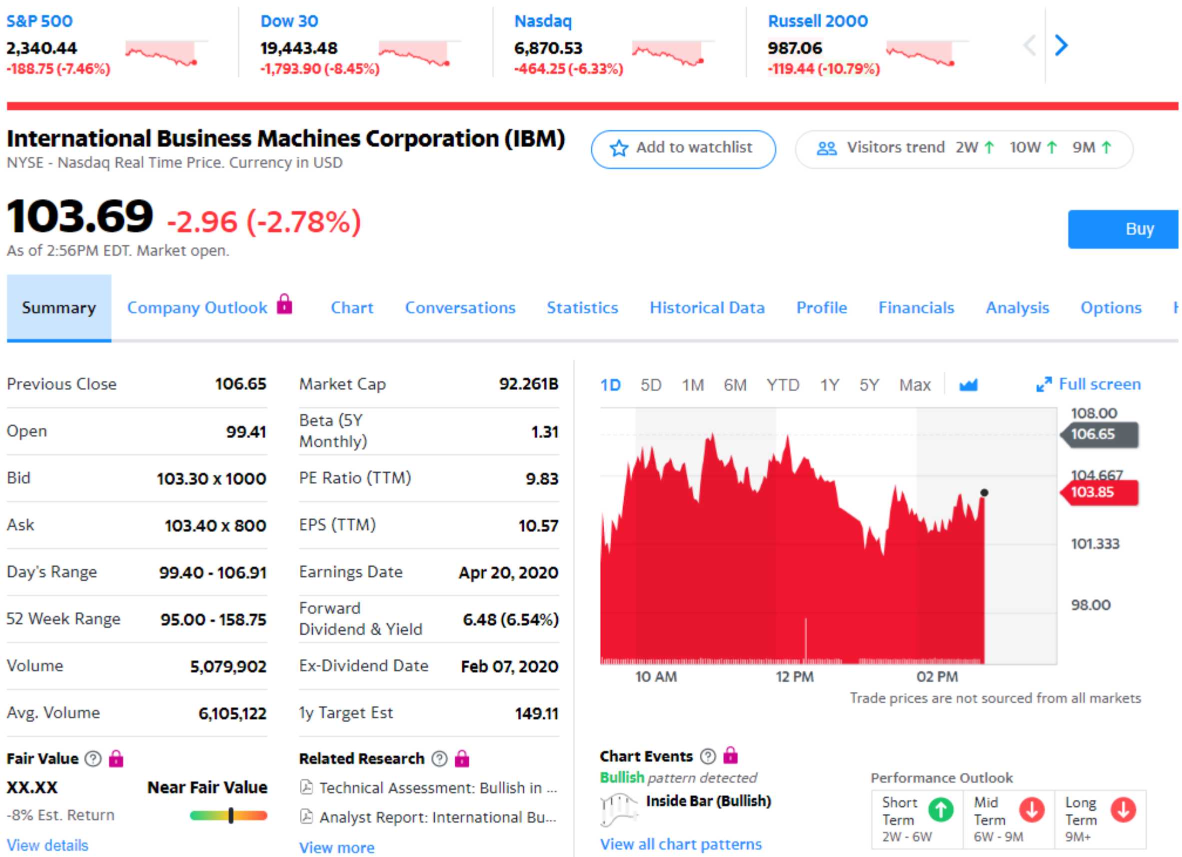
STOCK AND BOND VALUATION

It is well known that both bonds and stocks play a very important role in the financial markets. Because of this, we should learn how to price bond and stock, what is the impact of interest rate movement on the value of a bond, how to measure the risk of a bond, what is the definition of duration, what is the modified duration, what is so-called term-structure of interest rate and how to hedge the interest rate risk for bond buyers. For the stock valuation, we would learn the Dividend Discount Model and other widely used multiple methods. In particular, the following topics will be covered:

- What is a bond?
- What is a stock?
- Review of basic formulae: present value of one future cash flow, present value of annuity, present value of growing perpetuity
- Pricing model for a zero-coupon bond
- Pricing model for a coupon bond
- Term Structure of interest rate
- Duration measure and modified duration
- Relationship between bond price and YTM (Yield-to-Maturity)
- Excel `rate()` and `yield()` functions
- One-period and two-period models for stock pricing
- A general n-period model for stock pricing
- How to estimate a dividend growth rate

6.1 INTRODUCTION

For financial markets, bond and stocks are two most frequently traded securities. For example, on 3/18/2020, we could find the following image from Yahoo!Finance, http://finance.yahoo.com, for a given ticker symbol of IBM.

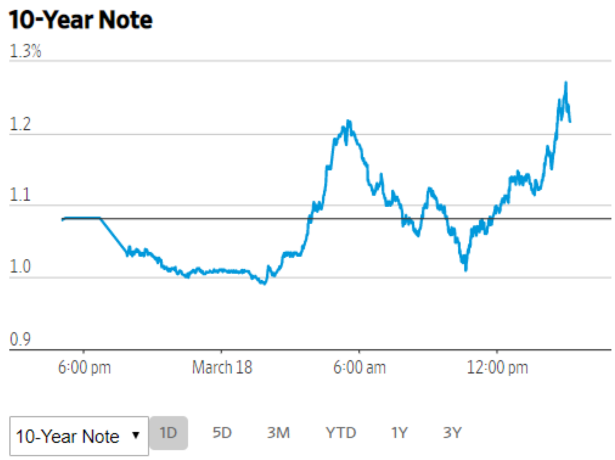


Based on it, if we plan to buy one share of the stock, the cost will be $103.40. If we sell one share instead, we would receive $130.3. The difference between bid and ask is called spread which is a good measure of liquidity. Here we ignore other potential fee involved. The 52-week's range is from $95.00 to $158.75. Later in the book, we would show a trading strategy is based on this 52-week range.

For the bond related information, we could try Wall Street Journal's Market Data Center (Bonds, Rates & Credit Markets) at http://www.wsj.com/mdc/public/page/mdc_bonds.html, see the image below.

U.S. Treasurys 3:10 PM EDT 3/18/20

	COUPON (%)	PRICE CHG	YIELD (%)	YIELD CHG
30-Year Bond	2	-3 26/32	1.834	0.143
10-Year Note	1.5	-1 3/32	1.216	0.134
7-Year Note	1.125	-7/32	1.091	0.101
5-Year Note	1.125	-3/32	0.798	0.056
3-Year Note	0.5	-1/32	0.637	0.016
2-Year Note	1.125	0/32	0.508	0.015
1-Year Bill	0	-3/32	0.176	-0.084
6-Month Bill	0	-5/32	0.084	-0.157
3-Month Bill	0	-5/32	0.018	-0.168
1-Month Bill	0	-2/32	0.025	-0.066



View Treasury Quotes Page

From the above image, on the left-hand sided, two columns are related to the term structure of interest rate (this topic will be discussed pretty soon). The first column is time, while the Yield column, the third one, is the YTM (Yield-to-Maturity). The relationship between those two is called the Term Structure of Interest Rate. Later in the chapter, we would show how important this so-called term structure of interest rate is.

6.2 WHAT ARE BONDS AND STOCKS?

A bond is a debt investment in which an investor lent money to an company government for a predefined period of time at a variable or fixed interest rate. In one sentence, a bond is quite similar to a loan. If the coupon rate is fixed, bond buyers would face certain risk, such as interest rate risk. Unlike bond, a stock is issued by a corporate as the part of the owner of the company. When the company issues their stocks the first time, it is called IPO (Initial Public Offer). Bond and stocks are quite different animals. Since bonds have fixed terms and future cash flows, many might think that analyzing bonds is easier than stocks. This is not true. Bonds are more difficulty to study.

6.3 WHICH IS MORE DIFFICULT TO ANALYSE?

First, both stock and bonds are difficult to analyses. However, one most asked question is: which security, bond or stock, is more difficult to analyses. Since bonds have fixed future cash flows and fixed time periods, most new learners would think that bonds are easier. Actually, the opposite is true: bonds are much more difficult to analyses. For this reason, we start with stock valuation.

6.4 FORMULA REVIEW

In Chapter 2: Time Value of Money, we have learnt a few formulae to estimate the present value of one future cash flow, present value of annuity, and present value of growing perpetuity. Since in this chapter, those formulae will be used intensively, we list them here one more time.

The formula to estimate the present value of one future cash flow is given below.

$$PV(one\ future\ cash\ flow) = \frac{FV}{(1+R)^n} \qquad (1)$$

where PV is the present value, FV is the future value or future cash flow, R is the effective period rate and n is the number of periods. The frequencies of R and n should be consistent. This means that if R is the effective annual (monthly/quarterly) rate, then R must be the number of years (months/quarters).

Annuity is defined as: the same cash flows, at the same interval for n periods, see two examples shown below.

```
                    C          C            C
|----------|----------|----------|---   ……  -|
0          1          2          3           n
```

Or

```
           C          C          C            C
|----------|----------|----------|---   ……  -|
0          1          2          3           n
```

when the first cash flow happens at the end of the first period, see the second case above, the present value of annuity is given below.

$$PV(annuity) = \frac{C}{R}\left[1 - \frac{1}{(1+R)^n}\right] \qquad (2)$$

where C is the repeated payment and n is the number of payments (periods).

If the cash flows grow at a constant rate of g forever, it is called growing perpetuity, see the time line and cash flows below.

```
           C      C(1+g)    C(1+g)^2    -> ∞
|----------|----------|----------|--------  ……
0          1          2          3          -> ∞
```

when the first cash flow of C happens at the end of the first period, and the rest cash flows grow at a constant rate of g, the present growing perpetuity is shown below.

$$PV(growing\ perpetuity) = \frac{C}{R-g} \qquad (3)$$

where C is the first cash flow and it happens at the end of the first period, R is the discount rate. Again, the frequencies of R and g should be constant. If R is the effective annual rate, then g must be annual growth rate.

On the other hand, if the first cash flow happens at the end of the k^{th} period, the following formula is used to estimate the present value of growing perpetuity.

$$PV(growing\ perpetuity) = \frac{C}{R-g}\frac{1}{(1+R)^{(k-1)}} \qquad (4)$$

where k is the time period when the first cash flow of C occurs. Obviously, when k takes a value of 1, Equation (4) would collapse to Equation (3)

6.5 ONE-PERIOD MODEL FOR STOCK PRICING

The simplest model for pricing stock is so-called one period model. Assume that after we receive the first dividend at the end of year 1, we would sell our stock at price p_1, see the following time line and the corresponding cash flows.

```
                                              d1+p1
|----------------------------------------------|
0                                              1
```

If the discount rate is R, the price would be given below.

$$P_0 = \frac{d_1+p_1}{(1+R)} = \frac{d_1}{(1+R)} + \frac{P_1}{(1+R)} \qquad (5)$$

Where P_0 is the stock price at time zero, i.e., today's price, d_1 is the first dividend as the end of the first period, p_1 is the selling price at the end of the first period. Here is an example. Assume that we could receive \$2 dividend at the end of the first year, the expected selling price of our stock is \$34. If the discount rate is 12%, what is the price we are willing to pay today? The answer is given below.

C2 =PV(C1,D2,E2)

	A	B	C	D	E
1		R=	0.12	0	
2		po=	(\$32.14)	1	36

The answer is the stock price today is \$32.14.

6.6 TWO-PERIOD MODEL FOR STOCK PRICING

For a two-period model, we have the following time-line and cash flow pattern.

```
                    d1                    d2+p2
|----------------------|------------------------|
0                      1                        2
```

$$P_0 = \frac{d_1}{(1+R)} + \frac{d_2}{(1+R)^2} + \frac{P_2}{(1+R)^2} \quad (6)$$

For a two-period model, we have the following time-line and cash flow pattern. Assume that we have \$1 and \$2 dividends at the ends of years 1 and 2. The expected selling price is \$58. What is the stock price today is the required return is 18%?

$$P_0 = \frac{1}{(1+0.18)} + \frac{2}{(1+0.18)^2} + \frac{58}{(1+0.18)^2} = 15.21$$

6.7 AN N-PERIOD MODEL FOR STOCK PRICING

We could generalize the above one-period and two-period models. For one-period model, we have one dividend plus a selling price. For a two-period, we have 2 future dividends plus a selling price. For an n-period model, we could have n future dividends plus a selling price. The selling price should be the summation of those n+1 future cash flows, see below.

$$P_0 = \frac{d_1}{(1+R)} + \frac{d_2}{(1+R)^2} + \cdots + \frac{d_n}{(1+R)^n} + \frac{P_n}{(1+R)^n} \quad (7)$$

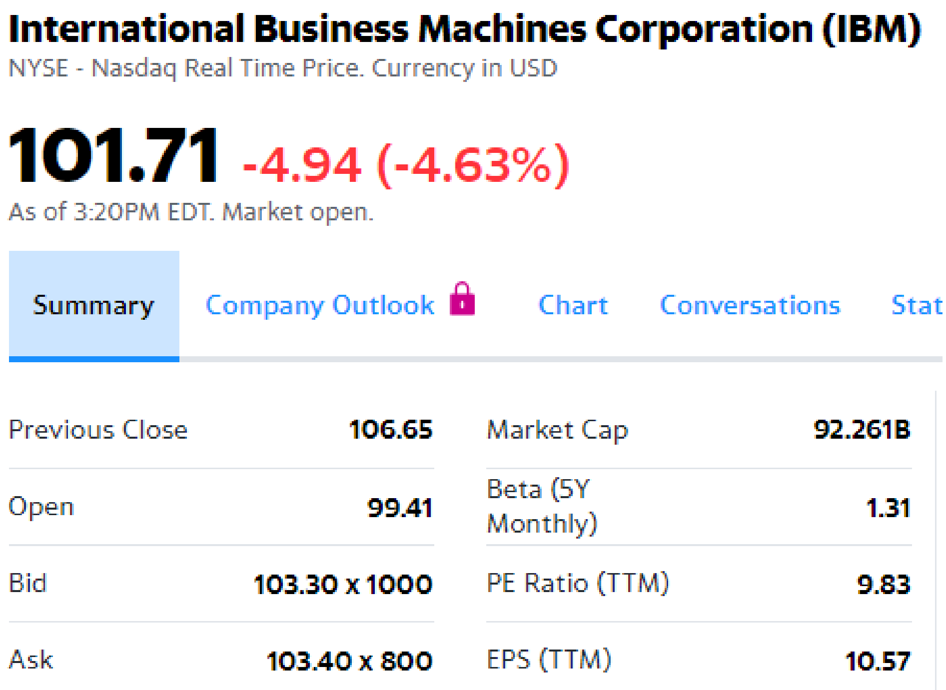
A new company expects to expand quickly during its first several years. To save money, the company would not distribute any dividend during the first 3 years. Starting year 4, the first dividend is expected as \$1. After that the dividend growth rate is 20% for the next 5 years. The long-term growth rate is expected to be 3% per year. If the required rate of return is 18% for this company, what might be its stock's selling price today? The result is shown below.

	A	B	C	D	E	F	G	H	I
1			year	div		total cash flow	pv		
2			0						
3			1	0		0	0.00	=PV(B8,C3,0,E3)	
4			2	0		0	0.00		
5			3	0		0	0.00		
6	g1	0.2	4	1		1	-0.52		
7	long-term g	0.03	5	1.20	=D6*(1+B6)	1.20	-0.52		
8	R	0.18	6	1.44		1.44	-0.53		
9			7	1.73		1.73	-0.54		
10			8	2.07		2.07	-0.55		
11	selling price	17.086464	9	2.49		19.57	-4.41		
12	formula	=D12/(B8-B7)	10	2.56	=D11*(1+B7)				
13						sum=price	-7.08	=SUM(F3:F11)	

First, we enter our initial information, such as the super growth rate of 20% and the long-term growth rate of 3%. For Column C, we have a "time line". The 5 red numbers are for 5-year growth with a 20% growth rate.

6.8 MULTIPLE TO ESTIMATE THE PRICE

At the beginning of the chapter, we have the following image for the IBM.



Since EPS is $10.57 and the latest closing price is $101.71, the PE ratio is 9.62. The reported is 9.83 shown above. Actually, if we have an industry average PE ratio, we could estimate the current price of a given stock if its EPS is known. For example, if the industry average PE ratio is 15. For a specific company we know that her EPS is $2 per share. Then we would argue that its price should be $30 (P/E *EPS = 15*2). Actually, we could apply other multiples such as P/EBIT, P/SALES, P/A and the like.

6.9 PRICING ZERO-COUPON BONDS

For a zero-coupon bond, investors would not receive any coupon payment over the life of the bond. On the maturity date, they would receive the face value. Actually, we could apply Equation (1): present value of one future cash flow.

The price formula for a zero-coupon bond is given below.

$$Price\ (zeroCoupon) = \frac{FV}{(1+YTM)^n} \tag{8}$$

Where Price is the bond price, FV is the face-value, YTM is the Yield-to-maturity while n is the number of years. For example, a zero-coupon bond has a face value of $1000 and will mature in 5 years. If the YTM is 3%, what is the selling price of this zero-coupon bond? The results are shown in the following image.

E3 | =PV(C2,C3,0,C1)

	A	B	C	D	E	F	G
1		FV	1000		862.6088	=C1/(1+C2)^C3	
2		YTM	0.03				
3		n	5		($862.61)	=PV(C2,C3,0,C1)	

e.

For this case, we apply the two ways to estimate the selling price: by apply Equation (5), see the formula contained in cell E3, or by calling the Excel PV() function, see the formula contained in cell E1.

6.10 PRICING COUPON BONDS

For a coupon bond, investors have two types of cash flows: coupon payments and the face value of the bond. Below, we show two examples. Example one: face value is $1,000, the annual coupon rate is 8% with a maturity of 5 years. If the coupon is paid once per year, the coupon payment is $80 (0.08*1000). Below are the cash flows of such a bond.

```
          80        80        80       80        80 +1000
|----------|----------|----------|---------|----------|
           1         2          3        4          5
```

Example #2: : the face value is $100, the coupon rate is 4% with a maturity of 3 years. The annual coupon payment is $4 (0.04*100) and the value of each coupon payment will be $2 (0.04*100/2), see the following formula for coupon payments.

$$C = \frac{FV*couponRate}{n} \tag{9}$$

Where C is the coupon payment, FV is the face value, `couponRate` is the coupon rate and n the how many times the investors would receive coupon payments. Below are the cash flows of such a bond.

```
      2      2      2     2      2      2 +100
|------|-------|------|------|------|------|
0              1             2             3
```

Obviously, the price of such a coupon bond would be the present value of those two types cash flows.

$$Price\ (coupon\ bond) = pv(coupon\ payments) + pv(face\ vaue)$$

More specifically, we have the following formula:

$$PV(bond) = \frac{C}{R}\left[1 - \frac{1}{(1+R)^n}\right] + \frac{FV}{(1+R)^n} \tag{10}$$

If use YTM instead of R, we would have the following formula.

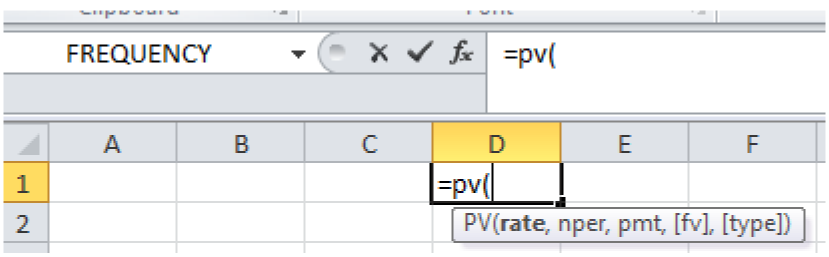
$$PV(bond) = \frac{C}{YTM/nper}\left[1 - \frac{1}{\left(1+\frac{YTM}{nper}\right)^n}\right] + \frac{FV}{(1+\frac{YTM}{nper})^n} \tag{11}$$

Where `nper` is the number of coupon payments. For annual coupon payment, `nper` is 1 and for a semi-annual coupon payment, `nper` is 2.

Here is the example. The face value of the bond is $1,000 with a maturity of 15 year. The coupon rate is 5% and paid semiannually. If the YTM is 4%, what is the selling price of the bond? The coupon payment is $25 every six months, n=30 (15*2) and R=2% (0.04/2), see the following estimation.

	A	B	C	D	E	F	G
1		FV	1000				
2		T (year)	15				
3		nper	2				
4		coupon rate	0.05				
5		YTM	0.04				
6							
7		n (=T * nper)	30	=C2*C3			
8		C	25	=C1*C4/C3			
9		R	0.02	=C5/C3			
10		price 1	1111.9823	=C8/C9*(1-1/(1+C9)^C7)+C1/(1+C9)^C7			
11		price2	($1,111.98)	=PV(C9,C7,C8,C1)			

The price 1 is based on the above formula while price 2 is based on the Excel function called `pv()`. Note that the input list of the `pv()` function is `=pv(rate,nper, pmt, [fv], [type])`, see below.



6.11 BOND PRICE VS. INTEREST RATE

The bond price is negatively correlated with interest rate. In other words, if the interest rate jumps, the bond price will fall. If the interest rate falls, the bond price will increase. The reason is given below. The bond price is the summation of the present value of the first cash flow, plus the present value of the second cash flow, and so on, see the following cash flow chart below.

$$bond\ price = \frac{c_1}{(1+R)} + \frac{c_2}{(1+R)^2} + \cdots + \frac{c_n}{(1+R)^n} + \frac{FV}{(1+R)^n} \quad (12)$$

Since the discount rate of R (YTM) is in the denominator, the higher this R (YTM) value the lower each ratio. Since every ratio (present value) is smaller, the net result is the price the bond will be smaller. The opposite is true: when we decrease the value of R (YTM), each present value will be higher than before, thus, the summation of all those present values will be higher. The correlation between price of the bond and YTM (R) is negative.

$$\rho(bondPrice, YTM) < 0 \quad (13)$$

where ρ is the correlation.

6.12 YTM, RATE() AND YIELD() FUNCTIONS

YTM is called yield-to-maturity which the return when a bond buyer holds his/her bond to maturity. We could use Excel `rate()` function to find out the YTM. The format of the function is given below

	A	B	C	D	E	F	G
1				=rate(			
2				RATE(**nper**, pmt, pv, [fv], [type], [guess])			

where `nper` is the number of coupon payments per year, `pmt` is the amount of each coupon payment, `pv` is the bond price, `fv` is the face value (or par value), `type` is for when coupon would receive, at the end of the period or beginning of the period and `guess` is what our estimate is.

Assume that our bond has a maturity of 10 years. The annual coupon rate is 3%, paid every six months. If the face value is $1,000 and purchasing price is $720 what is the YTM of this bond?

D2 fx =D1*C3

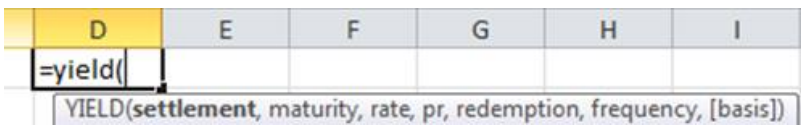
	A	B	C	D	E	F	G
1		face value	1000	3.37%	=RATE(C4*C3,C1*C2/C3,-C5,C1)		
2		coupon Rate	0.03	0.067455743	=D1*C3		
3		payments per year	2				
4		Maturity (years)	15				
5		price	650				

The YTM for the above bond is 6.746% per year. This YTM is quite similar to the APR with a semi-annual compounding frequency. The following table shows the relationship between price of the bond, in terms of face value and the coupon rate.

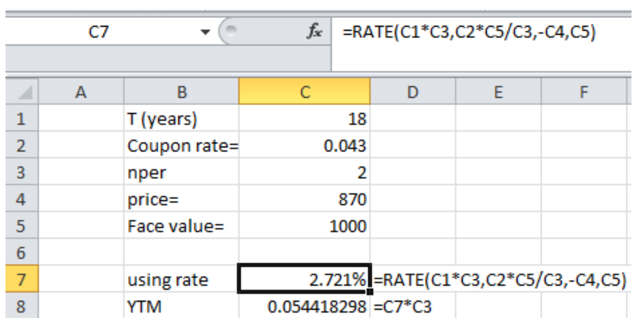
Table 6.1: Relationship between price of the bond and its face value

Condition	Price vs. Face value
When the coupon rate is higher than the discount rate	Price > Face value
When the coupon rate is the same as the discount rate	Price = Face value
When the coupon rate is lower than the than the discount rate	Price < Face value

Alternatively, we could use an Excel function called `yield()`. The following image shows its input variables.



The `settlement` is the starting day or purchasing day, `maturity` is when the bond would mature, `rate` is the annual coupon rate, `pr` is the selling price (present value of a bond) divided by the face value times 100, `redemption` is always 100, `frequency` is the number of times the coupon would be paid. Later, we would show how to call this function. The rate function is much easier to use and undertint than the `yield()` function. Below we show a simple comparison.


C7 =RATE(C1*C3,C2*C5/C3,-C4,C5)

	A	B	C	D	E	F
1		T (years)	18			
2		Coupon rate=	0.043			
3		nper	2			
4		price=	870			
5		Face value=	1000			
6						
7		using rate	2.721%	=RATE(C1*C3,C2*C5/C3,-C4,C5)		
8		YTM	0.054418298	=C7*C3		

On the other hand, when applying the Excel `yield()` function, we have to choose settlement and maturiy in a date format and pay attendion to the definitions of the selling price, `pr`, and `redemption` value, i.e., face value in our definition, see an example below.

C10 =YIELD(C6,C7,C2,C8,C9,C3)

	A	B	C	D	E	F	G
1		T (years)	18				
2		Coupon rate=	0.043				
3		nper	2				
4		price=	870				
5		Face value=	1000				
6		settlement date	1/1/2011	<= randomly choose oen day			
7		maturity date	1/1/2029	=DATE(YEAR(C6)+C1,MONTH(C6),DAY(C6))			
8		selling price	87.000	=C4/C5*100			
9		par value	100	<- always 100			
10		YTM	0.054418298	=YIELD(C6,C7,C2,C8,C9,C3)			

Compared with the previous result by using the Excel `rate()` function, we have the same result of 5.44% for the YTM estimation.

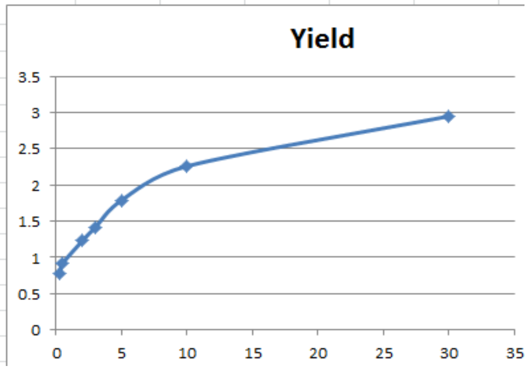
6.13 TERM STRUCTURE OF INTEREST RATE

For a bond investor, he/she would worry about the potential movement of the interest rate since we know from the previous sections that the bond price is negatively correlated with the interest rate. From the website at https://finance.yahoo.com/bonds, we could get the following information (on 9/28/2017).

	A	B	C	D	E
1	US Treasury Bonds Rates				
2	Maturity	Yield	Yesterday	Last Week	Last Month
3	3 Month	0.77	0.76	0.75	0.66
4	6 Month	0.92	0.92	0.92	0.73
5	2 Year	1.23	1.25	1.25	1.21
6	3 Year	1.42	1.44	1.46	1.47
7	5 Year	1.79	1.82	1.84	1.89
8	10 Year	2.26	2.3	2.31	2.37
9	30 Year	2.95	2.98	2.97	2.99

With the information of the first two columns, Maturity and Yield, we could easily draw a term structure of interest rate, see the image below.

Maturity	Yield
0.25	0.77
0.5	0.92
2	1.23
3	1.42
5	1.79
10	2.26
30	2.95



The above curve is an upward sloping curve which is a normal case. One theory argues that if the normal rate is the summation of real interest rate which is usually a constant and inflation, an upward sloping yield curve suggests that we would expect a higher inflation rate in the future.

6.14 CREDIT SPREAD

One of the major reason that we consider the usage of the term structure of inters rate is to estimate the discount rate of a corporate bonds. The difference between the YTM for a corporate bond and the YTM of a (central) government bond with the same maturity is called spread. Spread is a measure of default risk. The spread is an increasing function of maturity and is strongly correlated with the credit rating of the bonk or the firm. The following table shows the relationship between spread, credit rating and maturity.

6	Rating	1 yr	2 yr	3 yr	5 yr	7 yr	10 yr	30 yr
7	Aaa/AAA	5	8	12	18	28	42	65
8	Aa1/AA+	11.2	20	27	36.6	45.2	56.8	81.8
9	Aa2/AA	16.4	32.8	42.6	54.8	62.8	71.2	97.8
10	Aa3/AA-	21.6	38.6	48.6	59.8	67.4	75.2	99.2
11	A1/A+	26.2	44	54.2	64.6	71.4	78.4	100.2
12	A2/A	32.8	46.6	54.6	67	75.6	84.4	112.4
13	A3/A-	45.6	61.8	71.6	83.6	91.6	100.2	126
14	Baa1/BBB+	57.8	80	93	109.4	120	131.8	166.8
15	Baa2/BBB	47	95.2	109.4	127.4	139.4	151.8	190.8
16	Baa3/BBB-	95.4	120.4	134.8	153.2	165.2	178.2	217.8
17	Ba1/BB+	167.6	192.6	209	228.6	243.4	258.8	297.2
18	Ba2/BB	239.6	264.8	282.8	304.2	321.2	339.4	377.2
19	Ba3/BB-	311.8	337	357.2	379.6	399.4	420.2	456.6
20	B1/B+	383.6	409.6	431.4	455.6	477.6	500.8	536.2
21	B2/B	455.8	481.6	505.2	531	555.4	581.4	615.6
22	B3/B-	527.8	553.8	579.4	606.4	633.6	661.8	695.6
23	Caa/CCC+	600	626	653	682	712	743	775
24	US Treasury Yield	0.132	0.344	0.682	1.582	2.284	2.892	3.882

The unit is basis-point, except the last row. One basis point is one hundredth of 1%. For example, for an AA- rated bonk, if the maturity is 5-year, the spread is 59.8 which is 0.598%. For the last row, for year 5, the risk-free rate is 1.582%.

6.15 DURATION

For two zero-coupon bonds, it is obvious the bond with a longer-term is more risky than that short-term bond. How about coupon bonds? One of the good measure is called duration. The simple definition of bond's duration is how long (in terms of number of years) we could recoup our initial investment. Because of the unit of duration is the time or year, to be more specific. For zero-coupon bond, it is clear that the duration of zero-coupon bond is the same as its maturity.

$$D = T \tag{14}$$

Where D is the duration and T is the maturity. For example, a 5-year zero-coupon bond has 5-year duration. For multiple future cash flows, the duration is the weighted time, see the following formula.

$$D = \sum_{i=1}^{n} w_i T_i \tag{15}$$

Where D is the duration, n is the number of future cash flows, w_i is the weight and T_i is the time when the i^{th} cash flow happens. The above duration definition usually is called Macaulay. The Macaulay duration is the weighted average term to maturity of the cash flows from a bond. The weight of each cash flow is determined by dividing the present value of the cash flow by the price.
The weight, w_i, has the following formula.

$$w_i = \frac{pv(C_i)}{\sum_{i=1}^{n} pv(C_i)} \tag{16}$$

Where w_i is the weight of the i^{th} cash flow, pv(ci) is the present value of the i^{th} cash flow. In other words, a weight is the present value of the i^{th} cash flow divided by the present value of all cash flows. Since the present values of all cash flows will be the price of the bond, w_i could be re-written in the following way.

$$w_i = \frac{pv(C_i)}{BondPrice} \tag{17}$$

Next, we use one example to explain the concept of duration. Assume that we would receive $200 at the end of the second year, see the time line and cash flow below.

```
                                   200
|--------------------|------------------------|
                     1                        2
```

How long should we have to wait to recover our initial investment? Obviously, we have to wait for two years, i.e., the duration is 2.

Now, assume that we will receive $100 at the end of first year and another $100 at the end of the second year, see the cash flows pattern below.

100 100

|---------------------|-------------------------|

1 2

How long should we have to wait to recover our initial investment? If ignore the impact of the time value of money, our initial guess will be 1.5 year since we would recover $100 in year 1 and another $100 at the end of year 2. Let's call this first pass. Here is the second pass: since the first $100 is more valuable than the second one, we should have more weight for 1 than for 2. In other words, our second guess that we have weight less than 1.5 year, such as 1.48 years to recover our initial investment.

Now, let's assume that discount rate is 5% per year. In the following image, we apply the Excel pv() function to estimate the present values of those two future cash flows. The present value of the first $100 is $95.24, while the present value of the second $100 is $90.70. The total value is $185.94, see below.

G6 fx =SUM(G3:G4)

	A	B	C	D	E	F	G
1			Time	Cash flow	pv	weight	w(i)*T(i)
2	R=	0.05	0				
3			1	100	($95.24)	0.512195	0.512195
4			2	100	($90.70)	0.487805	0.97561
5				Total =	($185.94)		
6						D=	1.487805

Since the present value of the first cash flow is higher, it is understandable that its weight is higher as well, 51.22% (95.24/185.94), while the weight of the second $100 is 48.78%. The duration of those two cash flows is 1.4878 years, very close to our guess for the second pass.

Since the duration is defined how long we have to wait to recover our initial investment, the higher the D value is more risky of our bond portfolio. The duration of a bond portfolio will be the weighted individual bond's durations, as shown below.

$$D_p = \sum_{i=1}^{n} w_i D_i \qquad (18)$$

Where D_p is the duration of a bond portfolio, n is the number of the bond in the portfolio, w_i is the weight of the i^{th} bond, Di is the duration of the i^{th} bond. The w_i is defined below.

$$w_i = \frac{v_i}{\sum_{i=1}^{n} v_i} \qquad (19)$$

where v_i is the value of the first bond. Below we show one example.
Data Management Ltd has issued bonds with 15 years to maturity, a 5% annual coupon rate with a face value of $1,000. If the YTM is 8% per year and the coupon payment is paid twice per year, what is the current price? The following image shows the result.

G32 =SUM(G2:G31)

	A	B	C	D	E	F	G
1	maturty	15	time	cash flow	pv	weight	wi*Ti
2	face value	1000	0.5	25	($24.04)	0.032457	0.016229
3	coupon rate	0.05	1	25	($23.11)	0.031209	0.031209
4	YTM	0.08	1.5	25	($22.22)	0.030009	0.045013
5	coupon freq	2	2	25	($21.37)	0.028854	0.057709
6			2.5	25	($20.55)	0.027745	0.069361
7	bond price	($740.62)	3	25	($19.76)	0.026677	0.080032
8			3.5	25	($19.00)	0.025651	0.08978
30			14.5	25	($8.02)	0.010824	0.156944
31			15	1025	($316.03)	0.426706	6.400587
32					($740.62)	D=	9.9298

First, we generate two columns of "time" and "pv", see columns C and D. To make our image more manageable, rows between 8 and 30 are hidden. The summation of those present value will be the price of the bond, see cell E32. The column "weights" is the present value of each cash flow divided by the bond price. Finally the "wi*Ti" column is the contribution by each cash flow. The summation of all those products will be our duration which is 9.9298 years. In is quite difficult to calculate this way. In the following section, we show two simple ways to estimate the duration.

6.16 EXCEL DURATION FUNCTION

To estimate duration for a bond, we could apply the Excel function called `duration`. After type `duration(` , i.e., function name with a left parenthesis, to find out a list of input variables, see the image below.

D	E	F	G	H	I	J
	=duration(					

DURATION(**settlement**, maturity, coupon, yld, frequency, [basis])

There are 6 input values for the function:

1) `Settlement` is the starting day or any day, e.g., could be today, if it is not given
2) `Maturity` is when the bond would expire
3) `coupon` is the coupon rate
4) `yld` is the YTM (yield to maturity)
5) `frequency` is the number of coupons paid every year
6) `basis` will take a value of 0,1,,2,3 or 4 (default value is 0)

    0 for 30/360 i.e., 30 days in a month and 360 in a year

    1 for actual/actual

2 for actual/360
3 for actual/365
4 for European 30/360

For this book, we will ignore the last input variable without any meaningful impact. The related formula is given below.

$$D_{Mac} = \frac{\sum_{t=1}^{N} \frac{Pmt_t}{(1+YTM)^t} * t}{V_B} \tag{20}$$

where V_B is the value of the underlying bond. Here is an example. Data Management Ltd has issued bonds with 15 years to maturity, a 5% annual coupon rate with a face value of $1,000. If the YTM is 8% per year and the coupon payment is paid twice per year, what is the current price?

B6 =PV(B4/B5,B1*B5,B2*B3/2,B2)

	A	B	C	D	E	F
1	T=	15				
2	face value	1000				
3	coupon rate	0.05				
4	YRM	0.08				
5	frequency	2				
6	bond price	($740.62)				

For the duration, for many cases when the settlement is not specified, we could simpley assume any date for the settlement, e.g., today. Assume that the face value is $1,000 with 15 years to maturity. The annual coupon rate is 5% and the coupon will be paid twice per year. What is the duration when the YTM is 8%? The following image show how to apply the Excel `duration()` funciton.

D4 =DURATION(D1,D2,B3,B4,2)

	A	B	C	D	E	F	G	H
1	T=	15	settlement day	10/1/2017				
2	face value	1000	maturity date	10/1/2032	=DATE(YEAR(D1)+B1,MONTH(D1),DAY(D1))			
3	coupon rate	0.05						
4	YRM	0.08	duration	9.9297997	=DURATION(D1,D2,B3,B4,2)			
5	frequency	2						
6	bond price	($740.62)						

In the above illustration, since settlement is not specified we simply choose 10/1/2017. To find out the maturity date, we apply the Excel `date()`, see the formula in cell D2. For some cases, the Excel duration is quite helpful. For other cases, such as the above case the function is quite combersit. In Appendix, we show the VBA function called `durationYan`, see its application below.

C7 =durationYan(C1,C2,C3,C4,C5)

	A	B	C	D	E	F
1		YTM	0.08			
2		faceValue	1000			
3		coupon rate	0.05			
4		freq	2			
5		year	15			
6						
7		Duration	9.929799749	=durationYan(C1,C2,C3,C4,C5)		

6.17 DURATION, CHANGING IN YIELD AND CHANGE IN BOND PRICE

From the previous sections, we know that the bond price is negatively correlated with the yield change. For example, when the YTM increases, the rice of the bond would fall. The opposite is true. Then the yield falls, the price of the bond will increases. When the change in the yield is small, we have the following relationship between the percentage change in a bond price and yield change.

$$\frac{\Delta P}{P} = -\frac{D*\Delta YTM}{(1+\frac{YTM}{nper})} \tag{21}$$

Where P is the price of the bond, ΔP is the change in the bond price, D is the Macaulay duration (or simply duration), YTM is the yield-to-maturity, ΔYTM is the change in YTM, nper the compounding frequency. For example, if the coupon paid twice per year, n should have a value of 2. For example, if D is 8.5, change is YTM is 40 basis-point, compounding frequency is annual and the current YTM is 5%, then we could have a value of 0.03238095, (8.5*0.004/(1+0.05/1)). In other words, our bond value will fall by 3.24%.

The modify duration is defined below.

$$D_{modified} = -\frac{D}{(1+\frac{YTM}{n})} \tag{22}$$

Now, Equation (16) will be much easy to remember.

$$\frac{\Delta P}{P} = -D_{modified} * \Delta YTM \tag{23}$$

6.18 HEDGING: DURATION MATCHING

For a bank's depositors, they usually prefer short-term depositors for several reasons. First, long-term is more risky than the short-term. Second, they might need money in the short term such as rental payments, vacation, buying house or car or the other arrangements.

$$D_{obligations} = D_{assets} \tag{24}$$

In words, the bank should match the duration of liability with duration of assets.

SUMMARY

In this chapter, we have discussed many basic concepts related to how to price a bond and a stock. For the bond, we show that its current price is negatively correlation with the interest rate. The best measure of the riskiness of a bond is duration which is defined as the weighted time when we could recover our initial investment. For the stock pricing, we have discussed the Dividend Discount Model which essentially an n-period model.

In the next chapter, Chapter 7: CAPM, we could discuss how to use CAPM (Capital Asset Pricing Model) to estimate the market risk of a stock, the expected cost of equity, modified beta and how to estimate the beta of a portfolio and point out some shortcoming of the CAPM.

REFERENCES

http://www.wsj.com/mdc/public/page/mdc_bonds.html?refresh=on

FINRA, Corporate bond data, http://www.finra.org/industry/trace/corporate-bond-data

Appendix A: VBA for durationYan

There are 5 input variables: durationYan(YTM, faceValue, CouponRate, Fequency, maturityYears)

```
Function durationYan(YTM As Double, faceValue As Double, CouponRate As Double,
Frequency As Double, maturityYears As Double) As Variant
Dim i As Integer, rate As Double,coupon As Double, pv As Double, D As Double,
bondPrice As Double

bondPrice = 0
D = 0
coupon = faceValue * CouponRate / Frequency
rate = YTM / Frequency

For i = 1 To (Frequency * maturityYears)
           pv = coupon / (1 + rate) ^ i
           D = D + pv * 1 / Frequency * i
           bondPrice = bondPrice + pv
Next i
pv = faceValue / (1 + rate) ^ (i - 1)
D = D + pv * 1 / Frequency * (i - 1)
bondPrice = bondPrice + pv
durationYan = D / bondPrice

End Function
```

For example, =`durationYan(0.08,1000,0.05,2,15)` will give us a value of 9.929799749.
The above VBA could be downloaded at http://canisius.edu/~yany/excel/durationYan.txt.

Appendix B: Data Case #7: Fund raised from a new bond issue

Currently, you are working as a financial analyst at Apple Inc. (Nasdaq ticker symbol AAPL). The firm plans to issue 30-year corporate bonds with a total face value of $80 million in the United States. Each bond has a face value of $1,000. The annual coupon rate is 3.5%. The firm plans to pay coupon once every year at the end of each year. Your boss asked you to estimate how much fund the firm could raise today.

Answer the following three questions:

Q1: How much your company would receive today by issuing the above 30-year bonds?

Q2: What is the YTM (Yield to Maturity) of the bond?

Q3: How much extra money your company could receive if your company manages to increase its credit rating by one notch?

Basic concepts: 1) present value of a bond is the summation of all its discounted future cash flows.

$$PV(bond) = price = \frac{C_1}{(1+R_1)^1} + \frac{C_2}{(1+R_2)^2} + \cdots + \frac{C_{n-1}}{(1+R_{n-1})^{n-1}} + \frac{C_n+FV}{(1+R_n)^n} \quad (1)$$

2) find out the appropriate discount rate for each future cash flow.

$$R_i = R_{f,i} + S_i \quad (2)$$

where R_i is the discount rate for year i , $R_{f,i}$ is the risk-free rate, from the Government Treasury term structure of interest (yield curve) for year i and S_i the credit spread which depends on the credit raging of your firm. For example, if an investor buys one bond, he/she will expect $35 at the end of each year for the next 30 years. The present value of $35 at the end of 5th year will be $\frac{35}{(1+R_5)^5}$. Note that all those discount rates (R_1, R_2, R_3 and etc.) are expected to be different.

Step 1: Draw a time line of 30 years and mark all future cash flows. Hint #1: to simplify your notation, you could work with just one bond ($1,000 face value) first.

Step 2: Find $R_{f,i}$ for $i=1,2,3, .. 30$. Go to http://finance.yahoo.com/bonds (hint #2: you have to interpolate your values, see the next page).

Step 3: Find the relationship between spread (S_i) and the credit rating, *i=1,2, … 30. , see below.*

```
> x<-.show_bondSpread(0)
> .saveYan(x,"c:/temp/sprad.csv")
[1] "Your saved file is ==>c:/temp/sprad.csv"
>
```

Then, we could easily open the output by using Excel, see the image below.

	A	B	C	D	E	F	G	H
1	RATING	1YEAR	2YEAR	3YEAR	5YEAR	7YEAR	10YEAR	30YEAR
2	Aaa/AAA	5	8	12	18	28	42	65
3	Aa1/AA+	11.2	20	27	36.6	45.2	56.8	81.8
4	Aa2/AA	16.4	32.8	42.6	54.8	62.8	71.2	97.8
5	Aa3/AA-	21.6	38.6	48.6	59.8	67.4	75.2	99.2
6	A1/A+	26.2	44	54.2	64.6	71.4	78.4	100.2
7	A2/A	32.8	46.6	54.6	67	75.6	84.4	112.4
8	A3/A-	45.6	61.8	71.6	83.6	91.6	100.2	126
9	Baa1/BBB+	57.8	80	93	109.4	120	131.8	166.8
10	Baa2/BBB	47	95.2	109.4	127.4	139.4	151.8	190.8
11	Baa3/BBB-	95.4	120.4	134.8	153.2	165.2	178.2	217.8
12	Ba1/BB+	167.6	192.6	209	228.6	243.4	258.8	297.2
13	Ba2/BB	239.6	264.8	282.8	304.2	321.2	339.4	377.2
14	Ba3/BB-	311.8	337	357.2	379.6	399.4	420.2	456.6
15	B1/B+	383.6	409.6	431.4	455.6	477.6	500.8	536.2
16	B2/B	455.8	481.6	505.2	531	555.4	581.4	615.6
17	B3/B-	527.8	553.8	579.4	606.4	633.6	661.8	695.6
18	Caa/CCC+	600	626	653	682	712	743	775
19	US Treasury Yield	0.132	0.344	0.682	1.582	2.284	2.892	3.882

Except the last row (US Treasury Yield), the unit of spread is basis point which is one hundredth of 1%. For example, for the A rated bond, the spread is 67 bp which is the same as 0.67%. For the last row (US Treasury Yield), 0.132 is the same as 0.132%.

Step 4: Assume a rating of your company, such as A

Step 5: Apply the following formula to estimate the present value of each future cash flow.

$$PV_i = \frac{C_i}{(1+R_i)^i} \quad (3)$$

Step 6: Apply equation (1), i.e., take a summation.

Note: Linear interpolation.

First, let me use a simple example. Assume that the YTM for 5-year is 5%, the YTM for 10-year bond is 10%. What are the YTMs for 6, 7, 8, and 9-year bonds?

A quick answer is 6% for 6-year bond, 7% for 7-year bond, 8% for 8-year bond and 9% for 9-year bond. The basic idea is an equal incremental value.

Assume that YTM for a 5-year bond is R_5, YTM for a 10-year bond is R_{10}. Since there are 5 intervals between year 5 and year 10. Thus, the incremental value between each year is $\Delta = \frac{R_{10}-R_5}{5}$

For a 6-year bond, its value will be $R_5 + \Delta$
For a 7-year bond, its value will be $R_5 + 2\Delta$
For a 8-year bond, its value will be $R_5 + 3\Delta$
For a 9-year bond, its value will be $R_5 + 4\Delta$

Here is more detailed explanation.[1] If the two known points are given by the coordinates, (x_0, y_0) and (x_1,y_1), the linear interpolation is the straight line between these points. For a value *x* in the interval of (x_0,x_1), the value *y* along the straight line is given from the equation

$$\frac{y-y_0}{x-x_0} = \frac{y_1-y_0}{x_1-x_0} \quad (4)$$

which can be derived geometrically from the figure on the right. It is a special case of polynomial interpolation with *n* = 1.

Solving this equation for *y*, which is the unknown value at *x*, gives

$$y = y_0 + (x - x_0)\frac{y_1-y_0}{x_1-x_0} = y_0 + \frac{(x-x_0)y_1-(x-x_0)y_0}{x_1-x_0} \quad (5)$$

which is the formula for linear interpolation in the interval of (x_0, x_1).

Appendix C: Data Case #8: what is the dividend growth rate?

Based on the Dividend Discounted Model, we have the following n-step model,

$$P_0 = \frac{d_1}{(1+R)^1} + \frac{d_2}{(1+R)^2} + \cdots + \frac{d_n}{(1+R)^n} + \frac{P_n}{(1+R)^n} \quad (1)$$

where P_0 is the price today, d1 is the dividend #1 at the end of the first period, R is the discount rate, P_n is the selling price at the end of nth period. The value of selling is determined by the following formula.

$$P_n = \frac{d_{n+1}}{R-g} \quad (2)$$

[1] http://en.wikipedia.org/wiki/Linear_interpolation.

where d_{n+1} is the dividend at the end of n+1 period, g is an constant growth rate. R will be estimated in the next data case. For this data case, we try to estimate historical dividend growth rate.
Below is the procedure:

Step 1: go to the http://www.nasdaq.com/quotes/dividend-history.aspx
Step 2: enter a few tickers, such as ibm,msft, c and wmt
Step 3: retrieve historical dividend data,

IBM | MSFT | C | WMT

Save Stocks

The Dividend History page provides a single page to review all of the aggregated Dividend payment information. Visit our Dividend Calendar: Our partner, Zacks Investment Research, provides the upcoming ex-dividend dates for the next month.

Ex/Eff Date	Type	Cash Amount	Declaration Date	Record Date	Payment Date
8/8/2017	Cash	1.5	7/25/2017	8/10/2017	9/9/2017
5/8/2017	Cash	1.5	4/25/2017	5/10/2017	6/10/2017
2/8/2017	Cash	1.4	1/31/2017	2/10/2017	3/10/2017
11/8/2016	Cash	1.4	10/25/2016	11/10/2016	12/10/2016
8/8/2016	Cash	1.4	7/26/2016	8/10/2016	9/10/2016
5/6/2016	Cash	1.4	4/26/2016	5/10/2016	6/10/2016
2/8/2016	Cash	1.3	1/26/2016	2/10/2016	3/10/2016
11/6/2015	Cash	1.3	10/27/2015	11/10/2015	12/10/2015
8/6/2015	Cash	1.3	7/28/2015	8/10/2015	9/10/2015
5/6/2015	Cash	1.3	4/28/2015	5/8/2015	6/10/2015
2/6/2015	Cash	1.1	1/27/2015	2/10/2015	3/10/2015
11/6/2014	Cash	1.1	10/28/2014	11/10/2014	12/10/2014
8/6/2014	Cash	1.1	7/29/2014	8/8/2014	9/10/2014
5/7/2014	Cash	1.1	4/29/2014	5/9/2014	6/10/2014
2/6/2014	Cash	0.95	1/28/2014	2/10/2014	3/10/2014
11/6/2013	Cash	0.95	10/29/2013	11/8/2013	12/10/2013
8/7/2013	Cash	0.95	7/30/2013	8/9/2013	9/10/2013

Task: based on the historical dividend data, what are the annual dividend growth rates?

Comment your results.

EXERCISES

6.1 What security is more risky, bond or stock? Why?
6.2 What is the difference between zero-coupon bond and coupon bond?
6.3 With the same maturity, which one is more risky, zero-coupon bond and coupon bond?
6.4 With different maturities, which one is more risky, zero-coupon bond and coupon bond?
6.5 What is the definition of duration?
6.6 How to use simple sentences to explain the duration of a bond to a non-finance ordinal persons?
6.7 The face of bond is $1,000, the annual coupon rate is 5% with semi-annual coupon payment. If he YRM is 8%, what is the duration of this bond?
6.8 What the percentage bond price change for the above bond, if the YTM fall by 50 basis point. Please use two methods to estimate this value.
6.9 When estimate YTM for a bond, we could apply both Excel `rate()` and `yield()` functions, what are the differences between those two functions?
6.10 In the chapter, we have learnt the relationship between face-value, discount rate and coupon rate follow the following rules.

Relationship between price of the bond and its face value

Condition	Price vs. Face value
When the coupon rate is higher than the discount rate	Price > Face value
When the coupon rate is the same as the discount rate	Price = Face value
When the coupon rate is lower than the than the discount rate	Price < Face value

Use several examples show the above rules.

6.11 The selling price is $1,123 with a face value of $1,000. The bond would mature after 10 years. If the coupon rate is 4% paid twice every year. What is the YTM? Please apply both the Excel `rate()` and `yield()` functions.
6.12 What is modified duration? What it the relationship between (among) modified duration, percentage change in bond price and change in yield?
6.13 In the chapter, we have discussed two types of formula to estimate the price of band, see below.

$$PV(bond) = \frac{C}{R}\left[1 - \frac{1}{(1+R)^n}\right] + \frac{FV}{(1+R)^n} \quad (1)$$

and

$$PV(bond) = \frac{C}{YTM/nper}\left[1 - \frac{1}{\left(1+\frac{YTM}{nper}\right)^n}\right] + \frac{FV}{(1+\frac{YTM}{nper})^n} \quad (2)$$

Explain why those two formulae are the same.
6.14 What is the difference between one-period, 2-period and n-period model to price stock?

6.15 How important is the dividend growth rate in an n-period model when we try to price the price of a stock?
6.16 How to estimate the dividend's growth rate?
6.17 For given ticker, how to estimate the implied cost of equity?